بیرکاری

پۆلی شەشەمی وێژەیی

ژمارەی سپاردن ٣٣٣ ساڵی ٢٠٠٣

٢٧٠٣ کوردی - ٢٠٠٣ ز - ١٦٧٥٣ أ

چاپخانەی وەزارەتی پەروەردە - هەولێر

تیراژ (١٠٠٠) دانە

هەرێمی کوردستانی عێراق
وەزارەتی پەروەردە

ܚܟܘܡܬܐ ܕܟܘܪܕܣܬܢ - ܥܝܪܐܩ
ܘܙܝܪܘܬ ܡܪܕܘܬܐ

ܚܘܫܒܢܐ

ܣܕܪܐ ܬܠܝܬܝܐ ܡܨܥܝܐ

ܬܘܪܓܡܐ ܡܢ :
ܐܒܪܗܡ ܣܥܝܕ ܓܘܪܓܝܣ

ܛܒܥܐ ܩܕܡܝܐ
ܫ 6753 ܐ - 2003 ܡ

ܡܛܒܥܬܐ ܕܘܙܝܪܘܬ ܡܪܕܘܬܐ - ܐܪܒܝܠ

سەرپەرشتیارانی چاپ

نادر موشي مراد

اندريوس يوخنا كوركيس

ܦܘܫܩܐ ܩܕܡܝܐ

ܡܫܠܦܢܬܐ ܘܐܘܡܢܘܬܐ

ܫܥܬܐ (١ - ١)

ܪܫܘܡܐ ܕܡܫܘܚܬܐ :

ܡܢ ܒܬܪ ܪܫܡܐ ܕܕܪܓܬܐ ܕܡܫܘܚܬܐ ܕܝܠܗ ܝܠܘܦܐ
...ܬܐ. ܒܡܣܝܪܘܬܐ ܝܠܗܬܐ. ܘܒܝܠܦܣ ܒܗܢܐ ܦܘܫܩܐ ܚܕܡܐ ܟܒܝܪܐ ܕܡܫܘܚܬܐ
...ܒܝܪܐ ܡܢ ܡܫܘܚܬܐ ܕܦܘܠܚܢܐ ܡܟܬܒܢܐ ܕܫܥܝܐ ܕܡܫܘܚܬܐ ܠܒܘܪܐ.

...ܘܦܩܐ (١)

...ܕܠܗ ܒܡܫܦܘܬܐ ܕܝܘܡܢܐ ܐܠܡܐܪܘܬܢܐ ܒܪܫܡܢܬܐ ܝܠ ܗܘܝܐ ܕܐܪܒܥܐ
...ܡܢ ܩܠܝܒܘܢܐ. ܘܡܢ ܟܠ ܪܘܒܥܐ ܐܝܬ ܬܠܬܐ ܠܫܒܪܐ ܘܡܢ ܟܠ ܠܫܒܪܐ
... ܚܡܫܐ ܩܠܝܒܘܢܐ، ܗܟܢܐ ܝܠܗ ܡܝܬ̈ܐ ܕܩܠܝܒܘܢܐ ܠܫܥܬܐ.
...ܐ : ܡܚܙܝܬܐ ܝܠܗ ܠܫܥܬܐ ܕܪܘܐܠܐ ܒܪܘܒܥܐ ܕܐܬܐ :

ܒܘܢܬ̈ܐ (٦٠) ܩܠܝܒܘܢܐ

- ܪܘܒܥܐ ܩܕܡܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܫܘܪܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܡܨܥܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܐܚܪܝܐ
- ܪܘܒܥܐ ܬܪܝܢܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܫܘܪܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܡܨܥܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܐܚܪܝܐ
- ܪܘܒܥܐ ܬܠܝܬܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܫܘܪܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܡܨܥܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܐܚܪܝܐ
- ܪܘܒܥܐ ܪܒܝܥܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܫܘܪܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܡܨܥܝܐ
  - ...ܢ̈ܐ (٥) ܠܫܒܪܐ ܐܚܪܝܐ

ܗܕܐ ܘܕܝܢ :

ܡܢܝܢܐ ܕܡܠܒܘܫܘܬܐ = ٤ ٣ ٥ = ٦٠ ܡܠܒܘܫܘܬܐ

ܡܬܠܐ (٢)

ܡܦܩܕܠܗ ܡܠܟܐ ، ܚܕ ܡܢ ܡܠܦܢܐ ܕܐܣܟܘܠܐ ܕܩܪܝܬܐ ܕܐܝܬܠܗ ܬܪ ܗܘܟܡܐ ܡܢ ܐܣܟܘܠܐ ܟܕ ܐܝܬܠܗ ܠܗ ܡܥܕܪܢܘ (٥) ܡܕܪܫܝܐ ܘܡܢ ܟܠ ܡܕܪܫܝܐ ܐܝܬ (٩) ܡܬܘܠܡܕܐ ܦܪܝܕܐ ܘܡܢ ܟܠ ܡܬܘܠܡܕܬܐ ܐܝܬ (٦) ܠܘܬܐ ܦܪܝܕܐ . ܟܡܐ ܐܝܠܗ ܡܢܝܢܐ ܕܐܣܟܘܠܐ ܕܐܝܬ ܠܗ ܡܥܕܪܢܐ .

ܦܢܝܬܐ ܐܝܠܗ ܟܠܝܬܐ ܕܡܬܠܐ ܒܡܘܕܥܐ ܐܝܟ ܠܗ ܡܬܠܐ ܩܕܡܝܐ ܘܒܗܘܝ ܟܠܝܐ ܕ:

ܡܢܝܢܐ ܕܐܣܟܘܠܐ = ٥×٩×٦ = ٢٧٠ ܐܣܟܘܠܐ

ܘܡܟܠܝܣ ܠܗ ܚܘܫܒܐ ܬܪܝܢ ܕܐܝܬ ܟܠܝܬܐ. ܘܟܠܝܬܐ ܐܝܠܗ ܕܬܪܝܢ ܡܘܦܠܝܐ ܠܗ ܚܕܐ ܕܐܕܪܫ ܡܬܠܐ ܐܝܠܗ ܚܕ.

ܘܟܠ ܕܐܗܐ ܦܢܝܬܐ ܐܝܠܗ ܒܩܠܝܬܐ ܕܡܐܡܪܐ ܩܕܡܝܐ ܕܐܬܐ ܕܦܪܘܫܐ ܐܝܠܗ ܠܟܐ ܡܘܦܠܝܠܗ ܠܗ ܒܬܪܝܬܐ ܕܐܕܪܫ ܡܬܠܐ ܚܒܝܪܐ.

<u>ܡܐܡܪܐ ܩܕܡܝܐ (١ - ١) (ܡܐܡܪܐ ܩܕܡܝܐ ܠܡܢܝܢܐ)</u>

ܐܢ ܣܝܡܠܢ ܕܐܝܬܠܢ ܡܢܝܢܐ ܡܢ ܫܘܟܠܠܘܬܐ (ܓܢܒܝܬܐ) ܒܘܟܠܝܗ (ܢ)
ܡܢ ܫܘܟܠܠܘܬܐ ܘܗܘܐܠܗ ܦܢܝܬܐ ܡܢܝܬܐ ܒܫܘܟܠܠܘܬܐ ܩܕܡܝܬܐ
ܒܡܢܝܢܐ ܡܢ ܐܘܪܚܬܐ ܒܘܟܠܝܗ (ܢ١) ܘܗܘܐܠܗ ܦܢܝܬܐ ܡܢܝܬܐ
ܒܫܘܟܠܠܘܬܐ ܬܪܝܢܝܬܐ ܒܡܢܝܢܐ ܡܢ ܐܘܪܚܬܐ ܒܘܟܠܝܗ (ܢ ٢)
ܘܫܘܟܠܠܘܬܐ ܬܠܝܬܝܬܐ ܒܡܢܝܢܐ ܡܢ ܐܘܪܚܬܐ ܒܘܟܠܝܗ (ܢ٣) ...
ܘܫܘܟܠܠܘܬܐ ܡܢ ܕܪܓܐ (ܢ) ܒܡܢܝܢܐ ܡܢ ܐܘܪܚܬܐ ܒܘܟܠܝܗ (ܢ ܢ) ،
ܐܝܟܐ ܒܟܒܪܐ ܕܚܕܐ ܫܘܟܠܠܘܬܐ ܠܐ ܡܥܒܕܐ ܥܠ ܒܟܒܪܐ ܕܐܦܚܕܐ
ܡܢ ܫܘܟܠܠܘܬܐ ܐܚܪܢܐ ، ܘܐܝܬ ܡܢܝܢܐ ܒܘܟܠܗ (ܢ١ × ܢ ٢ × ܢ ٣ ×
ܢ ܢ) ܡܢ ܦܠܚܐ ܡܫܡܠܝܐ ܐܝܟܢ ܕܒܬܪ ܟܠܝܗ ܫܘܟܠܠܘܬܐ (ܐܘ
ܓܢܒܝܬܐ) ܕܡܢܝܢܗ (ܢ)

ܡܘܦܩܬܐ (٣)

ܐܢ ܗܘܐ ܠܢ ܪ̈ܡܙܐ ١ ، ٢ ، ٣ ، ٤ ، ٥ . ܟܡܐ ܡܢܝܢ̈ܐ ܪ̈ܡܙܘܗܝ ܬܪ̈ܝܢ ܡܢ ܗܪ̈ܟ
ܪ̈ܡܙܐ ܡܨܝܬܐ ܝܠܗ ܡܫܟܚܬܐ ܢܒܠܗ ܐܢܝܢ : (ܐ) ܒܥܠܡ ܒܫܘܚܠܦܐ ܕܪ̈ܡܙܐ
ܠܐ ܡܢܝܢ̈ܐ ܒܓܘܗ (ܒ) ܠܐ ܒܥܠܡ ܒܫܘܚܠܦܐ ܕܪ̈ܡܙܐ ܠܐ ܡܢܝܢ̈ܐ ܒܓܘܗ .
ܠܠܒܬܐ ܝܠܗ ܕ : ܡܢܝܢ̈ܐ ܕܠܚܘܕܝ̈ܐ ܕܪ̈ܡܙܐ ܡܕܡܟܢܐ = ٥
ܡܢܝܢ̈ܐ ܕܠܚܘܕܝ̈ܐ ܕܪ̈ܡܙܐ ܬܪ̈ܝܢܐ = ٥
ܘܒܡܦܠܚܬܐ ܕܡܬܡܪܐ ܡܕܡܟܢܐ ܠܡܢܝܢܬܐ ܒܡܢܝܢ ܝܠܗ :
ܡܢܝܢ̈ܐ ܕܡܢܝܢ ܒܓܘܗ ܐܝܬ ܐܚܝܕܘܬܐ = ٥ × ٥ = ٢٥ ܡܢܝܢ̈ܐ
(ܒ) ܐܢܝܢ ܕܠܐ ܦܫܬܐ ܒܥܝܢܬܐ ܒܫܘܚܠܦܐ ܕܪ̈ܡܙܐ ܠܐ ܡܢܝܢ̈ܐ ܒܓܘܗ.
ܠܠܒܬܐ ܝܠܗ : ܡܢܝܢ̈ܐ ܕܠܚܘܕܝ̈ܐ ܕܪ̈ܡܙܐ ܡܕܡܟܢܐ = ٥
ܡܢܝܢ̈ܐ ܕܠܚܘܕܝ̈ܐ ܕܪ̈ܡܙܐ ܬܪ̈ܝܢܐ = ٤
ܘܒܡܦܠܚܬܐ ܕܡܬܡܪܐ ܡܕܡܟܢܐ ܠܡܢܝܢܬܐ ܒܡܢܝܢ ܝܠܗ :
ܡܢܝܢ̈ܐ ܕܡܢܝܢ ܒܓܘܗ ܐܝܬ ܐܚܝܕܘܬܐ = ٥ × ٤ = ٢٠ ܡܢܝܢ̈ܐ .

ܡܘܦܩܬܐ (٤)

ܟܡܐ ܡܢܝܢ̈ܐ ܪ̈ܡܘܙܐ ܡܬܘܪܐ ܝܠܗ ܡܢ (٣) ܬܠܬܐ ܘܒܨܘܪܐ ܡܢ (٤٠٠)
ܡܨܝܢܬܐ ܝܠܗ ܡܬܪܨܬܗ ܒܡܦܠܚܬܐ ܕܪ̈ܡܙܐ ٢ ، ٣ ، ٤ ، ٥ ، ٦ ، ٧
ܐܝ ܬܢܝܬܐ ܕܪ̈ܡܙܐ ܠܐ ܡܢܝܢ̈ܐ ܒܓܘܗ ܠܐ ܗܘܝܐ .

ܬܪܝܐ :

ܠܠܒܬܐ ܝܠܗ ܕܡܢܝܢܐ ܒܨܘܪܐ ܡܢ (٤٠٠) ܗܘܐ ܪ̈ܡܙܐ ܕܐܡܝܐ ܠܦܘܫ
ܒܨܘܪܐ ܡܢ (٤) ܘܟܠ ܕܗܘܐ :
ܡܢܝܢ̈ܐ ܕܠܚܘܕܝ̈ܐ ܕܪ̈ܡܙܐ ܕܐܡܝܐ = ٢
ܡܢܝܢ̈ܐ ܕܠܚܘܕܝ̈ܐ ܕܪ̈ܡܙܐ ܕܠܡܐܬܐ = ٥ (ܚܘܪ ܠܪ̈ܡܙܐ ܠܐ ܡܘܦܩܬܐ)
ܡܢܝܢ̈ܐ ܕܠܚܘܕܝ̈ܐ ܕܪ̈ܡܙܐ ܕܚܕܝܐ = ٤
ܘܒܡܦܠܚܬܐ ܕܡܬܡܪܐ ܡܕܡܟܢܐ ܠܡܢܝܢܬܐ ܒܡܢܝܢ ܝܠܗ :
ܡܢܝܢ̈ܐ ܕܡܢܝܢ = ٢ × ٥ × ٤ = ٤٠ ܡܢܝܢ̈ܐ

ܕܘܪܫܐ (١ - ١)

ܫ١ : ܠܘܬ ܢܝܢܘܗ (٤) ܡܚܘܬܐ ، (٥) ܢܗܪܘܬܐ ، (٨) ܣܘܪܓܕܐ . ܟܡܐ ܐܘܪܚܬܐ
ܐܝܬ ܕܓܒܝܢ ܚܕ ܢܝܢܘܗ ܠܡܒܝܢܬܐ ܡܬܦܪܫܬܐ ܡܢ ܡܚܘܬܐ ܘܢܗܪܘܬܐ
ܘܣܘܪܓܕܐ ؟

ܫ٢ : ܟܡܐ ܫܘܚܠܦܐ ܡܬܦܪܫܝܢ ܡܢ ܐܪܒܥܐ ܐܬܘܬܐ ܡܫܚܠܦܬܐ ܐܝܟ ܡܬܬܪܨܝܢ ܡܢ
ܐܬܘܬܐ :

ܐ ـ ܒ ـ ܓ ـ ܕ ـ ܗ ـ ܘ ـ ܙ ، ܐܢܗܝ ܕܐܬܘܬܐ ܠܐ ܫܚܠܦܐ ܒܢܝܢܐ ܠܐ ܫܪܝܐ .

ܫ٣ : ܟܡܐ ܡܢܝܢܐ ܡܬܦܪܫܝܢ ܐܝܟ ܕܡܬܚܫܒ ܡܢ (٣) ܡܢܝܢܐ ܘܪܒ ܡܢ (٧٠٠) ܡܫܚܠܦܬܐ
ܐܝܟ ܕܡܬܚܫܒ ܒܡܬܦܠܓܘܬܐ ܕܡܢܝܢܐ ٤ ، ٥ ، ٦ ، ٧ ، ٨ ، ٩ ܐܢ ܗܘܐ
(ܐ) ܐܢܗܝ ܕܡܢܝܢܐ ܠܐ ܡܢܝܢܐ ܒܢܝܢܐ ܫܪܝܐ .
(ܒ) ܐܢܗܝ ܕܡܢܝܢܐ ܠܐ ܡܢܝܢܐ ܒܢܝܢܐ ܠܐ ܫܪܝܐ .

ܡܫܚܠܦܢܘܬܐ :

ܗܐ ܓܠܝܬܐ ܕܪ̈ܫܐ ܕܡܫܚܠܦܢܘܬܐ ܒܢܘܚܣ ܠܡܘܦܬ ܕܐܬܐ :

ܡܬܠܐ (١)

ܐܢ ܗܘܐ ܐ = { ١ ، ٢ ، ٣ } ܟܡܐ ܡܫܚܠܦܢܘܬܐ ܒܪܘܟܒܬܐ ܡܨܝܢܐ
ܕܢܗܘܘܢ ܡܢ ܡܢܝܢܐ ܐ ܠܡܢܝܢܐ ܐ . ܒܪܘܟܒܬܐ ܡܢ ܡܢܝܢܐ
ܐ ܠܡܢܝܢܐ ܕܢܗܘܘܢ ܐܝܟ ܒܐܣܟܝܡܐ ܕܐܬܐ :

ܐ ܐ ܐ ܐ ܐ ܐ

(٣) (٢) (١)

ܐ ܐ ܐ ܐ ܐ ܐ

... ܗܘܐ ܕܐܝܬ ܐܝܬܐ ܡܫܚܠܦܢܘܬܐ ܕܢܗܘܘܢ ܒܪܘܟܒܬܐ ܡܢ ܡܢܝܢܐ ܐ ܐ
... ܐ ܐ ܘܡܨܝܢܐ ܠܗ ܡܬܚܫܒܐ ܕܡܢܝܢܐ ܕܒܪܘܟܒܬܐ ܒܡܫܚܠܦܢܘܬܐ
... ܕܡܢܝܢܐ ܘܗܘܐ ܐܝܟ ܕܐܬܐ :
... ܠܡܫܘܚܬܐ (١) ܐܝܬ ܬܠܬ ܬܠܬܐ ܨܘܪ̈ܬܐ ܡܬܡܨܝܢ ܠܗ ܕܢܗܘܘܢ
... ٣ ، ٢ ،
... ܠܡܫܘܚܬܐ (٢) ܐܝܬ ܬܪܬܝܢ ܨܘܪ̈ܬܐ ܡܬܡܨܝܢ ܠܗ (ܚܕ ܨܘܪܬܐ
... ܕܡܠܘ ܓܒܝܬܐ ܠܡܫܘܚܬܐ (١) ܠܐ ܗܠܘ ܡܨܝܢܐ ܓܒܝܬܐ ܢܣܒ ܡܢ ܚܕ
... ܐ ܒܗܝ ܡܫܚܠܦܢܘܬܐ ܗܠܝܢ ܒܪܘܟܒܬܐ).
... ܠܡܫܘܚܬܐ (٣) ܐܝܬ ܠܗ ܚܕܐ ܨܘܪܬܐ ܠܬܠܬܐ ܡܢܝܢܐ ܒܬܪܝܢܐ . ܘܟܠ
... ܗܘܐ ܡܢܝܢܐ ܕܒܪܘܟܒܬܐ = ٣ × ٢ × ١ = ٦ ܒܪܘܟܒܬܐ .
... ܐ ܒܬܪܝܢ ܐܝ ܗܘܬ ܐ ={ ١ ، ٢ ، ٣ ، ٤ } ܒܗܘܐ :
... ܡܢܝܢܐ ܕܒܪܘܟܒܬܐ ܡܢ ܐ ܠ ܐ = ٤ × ٣ × ٢ × ١ = ٢٤ ܒܪܘܟܒܬܐ.
... ܨܘܪܬܐ ܓܘܢܝܐ ܐܝ ܡܢܝܢܐ ܕܡܫܘܚܬܐ ܕܡܢܝܢܐ ܐ ܐܝܬ ܗܘܬ ، ܚܕ ، ܡܢܝܢܐ
... ܡܬܚܙܐ. ܡܢܝܢܐ ܕܒܪܘܟܒܬܐ ܡܢ ܐ ܠ ܐ = ܢ ، (ܢ - ١) (ܢ - ٢) ... × ١

ܡܫܪܪܬܐ (١ - ٢ - ١)

ܐ، ܐ ܝܗܘܐ ܚܘܒܫܬܐ ܫܠܡܬܐ ، ܟܠ ܡܦܠܣܬܐ ܕܩܕܡܘܟܠܘܬܐ
ܡܢ ܚܘܒܫܬܐ ܐ ܠܚܘܒܫܬܐ ܐ ܦܫܛܐ ܡܕܝܢܬܐ ܡܫܠܡܬܐ ܥܠ
ܚܘܒܫܬܐ ܐ.

ܗܦ ܡܢܝܢܐ ܗܠܘ ܣܓܝܐܐ ܕܡܫܬܠܡܢܝܬܐ ܚܘܒܫܬܐ ܫܠܡܬܐ ܡܩܕܡܐ ܠܟܠܢܝܗ ܡܕܒܪܘܬܐ ܡܫܡܪܢܐ ܠܐܛܘܦܣܐ ܕܐܗܐ ܚܘܒܫܬܐ (ܚܘܪ ܛܘܦܣܐ (١) ܠܥܠ)

ܡܫܪܪܬܐ (١ - ٢ - ٢)

ܠܟܠ ܢ، ܡܢܝ̈ܢܐ ܟܝ̈ܢܝܐ ܦܝܫ ܡܕܪܐ ܦܠܛܐ ܕܚܘܦܦܐ
ܢ(ܢ − ١)(ܢ −٢) ... × ٣ × ٢ × ١ ܚܘܫܒܐ ܕܡܢܝ̈ܢܐ (ܢ) ܘܦܝܫ
ܪܫܝܡܐ ܒܪܘܫܡܐ ⌊ܢ ܘܡܬܠ ܕܐܗܐ

⌊ܢ = ܢ! = { ܢ(ܢ − ١)(ܢ − ٢) ... × ٢ × ١ ܐܝܢ ܢ ≠ ܨܦܪܐ ; ١ ܐܝܢ ܢ = ܨܦܪܐ }

ܘܡܢ ܗܪܐ ܕܦܝܫܠܗ ܕܝܕܥܐ ܗܘܐ ܡܢܝ̈ܢܐ ܕܡܫܬܠܡܢܝܬܐ ܚܘܒܫܬܐ
ܫܠܡܬܐ ܡܢܝ̈ܢܐ ܕܐܛܘܦܣܐ، ܗܘܐ ⌊ܢ

ܛܘܦܣܐ (٢)

((ܐ)) ⌊٦ = ١ × ٢ × ٣ × ٤ × ٥ × ٦ = ٢٧٠

((ܒ)) ⌊١٠ = ١ × ٢ × ٣ × ... × ٨ × ٩ × ١٠ = ٣٦٢٨٨٠٠

ܛܘܦܣܐ (٣)

ܗܘܟܐ ܕܦܘܠܚܢܐ ܡܫܘܕܥܐ ܡܢ (١٠) ܦܪܨܘܦܐ، ܦܝܫܠܗ
ܛܠܝܒܬܐ ܦܠܓܗ ܥܠ ܟܪܣܝܐ ܦܪܝܫܐ ܐܝܟܐ ܕܟܠ ܦܪܨܘܦܐ ܡܢܐ
ܥܠ ܦܘܪܫܐ ܟܘܪܐ ܗܘ ܡܢܬܐ ܕܦܘܠܚܢܐ ܥܡ ܒܪܟܬܐ ܕܡܢܝ̈ܢܐ
ܕܟܪܣܝܐ (١٠) ܟܪܣܝܐ ܦܪܝܫܬܐ. ܟܡܐ ܗܠܘ ܡܢܝ̈ܢܐ ܕܐܘܪܚܬܐ
ܕܡܨܝܢܬܐ ܗܠܘ ܟܝܫ ܦܠܓܬܐ ܕܐܗܐ ܗܘܟܐ.

ܡܢܝ̈ܢܐ ܕܐܘܪܚܬܐ = ⌊١٠ =
١ × ٢ × ٣ × ... × ٨ × ٩ × ١٠ = ٣٦٢٨٨٠٠

ܛܘܦܣܐ (٤)

ܐܝܬܐ ܦܪܨܘܦܐ ܕܠܝܐ ܠܝܗ ܟܠ ܚܕܐ ܗܪܟܐ . ܒܟܡܐ
ܡܫܚܠܦܘܬܐ ܦܪܨܘܦܐ ܡܨܝܬܐ ܠܗ ܕܠܝܚܗ.
ܡܢܝܢܐ ܕܡܫܚܠܦܘܬܐ = ٦| = ٦ × ٥ × ٤ × ٣ × ٢ × ١ = ٧٢٠

ܪܘܬܐ

ܡܕܟܪܬܐ (١ ـ ٢ ـ ٣)
ܐ. ܚܘܫܒܐ ܕܗܘܝܐ ܠܡܢܝܢܐ ܡܢܝܢܐ ܕܐܫܚܠܦܘܬܗ ، ܒ
ܕܗܘܝܐ ܡܢܝܢܐ ܡܢ ܐ ܡܢܝܢܐ ܕܐܫܚܠܦܘܬܗ ܪ . ܟܠ
ܡܚܠܦܬܐ ܡܫܚܠܦܐ ܡܢ ܕܗܘܝܐ ܒ ܠܕܗܘܝܐ ܐ ܦܢܐ ܡܢܝܬܐ
ܡܫܚܠܦܬܐ ܐܘ ܡܫܚܠܦܢܐ ܠܐܫܚܠܦܘܬܐ ܡܢܝܢܗ . ܒܡܬܠ ܪܒ
ܪܒ ܘܦܢܝܐ ܪܡܝܘܬܐ ܠ(ܐ ، ܪ) ܐܘ ܠ ܘܦܢܝܐ ܡܢܝܬܐ
ܡܫܚܠܦܢܬܐ ، ܒܡܬܠ ܪܒ ܪܒ.

ܘܦܬܐ (٥)
ܐܝܬ ܡܢܝ ܟܠ (١٥) ܦܬܩܐ ܡܪܫܡܐ ܡܢ ١ ، ٢ ، ٣ ، ... ، ١٤ ،
١٥ ܐܘ. ܠܢܩܠ (٣) ܦܬܩܐ ܦܪܝܫܐ ܕܠܐ ܡܫܚܠܦܢܗ. ܒܟܡܐ ܦܠܓܐ
ܦܪܝܫܐ ܡܨܝܬܐ ܠܗ ܕܦܠܛ ܡܢ ܕܐܗܐ ܠܢܩܠܐ.

ܪܝܐ :
ܠܠܝܐ ܠܗ ܕ : ܦܬܩܐ ܩܕܡܝܐ ܕܢܩܠ ܟܠ ܚܕܐ ܗܘܐ ܡܢܝܢܐ
ܕܠܒܝܠܐ ܠܗ = ١٥
ܘܡܢܝܢܐ ܕܠܒܝܠܐ ܕܦܬܩܐ ܬܪܝܢܐ ܕܢܩܠ ܚܕܐ ܡܫܚܠܦܐ
ܠܚܕܐ ܕܦܬܩܐ ܩܕܡܝܐ = ١٤
ܘܡܢܝܢܐ ܕܠܒܝܠܐ ܦܬܩܐ ܬܠܝܬܝܐ ܕܢܩܠ ܚܕܐ ܡܫܚܠܦܐ
ܠܚܕܢܝܗ ܚܕܐ ܕܦܬܩܐ ܬܪܝܢܐ = ١٣
ܘܡܚܠܦܬܐ ܕܡܫܚܠܦܐ ܩܕܡܝܐ ܠܡܢܝܢܐ ܒܗܘܐ
ܡܢܝܢܐ ܕܦܠܓܐ = ١٥ × ١٤ × ١٣ = ٢٧٣٠

ܘܦܬܐ (٦)
ܡܘܬܒܐ ܕܡܫܚܠܦܘܬܐ ܕܚܕܐ ܒܘܬܦܘܬܐ ܡܬܘܬܪܐ ܠܗ ܡܢ
(١١) ܗܘܡܐ . ܒܟܡܐ ܐܘܪܚܬܐ ܡܨܝܬܐ ܠܗ ܠܒܝܬܐ ܕܠܬܐ
ܦܪܨܘܦܐ ܡܢܝܗ ܗܘ ܒܡܠܬܐ ܕܕܪܓܐ ܕܪܫܐ ܘܫܠܟ ܪܒܢܐ ܘܢܗܪܐ

ܐܝܟ ܗܟܢ ܝܕܥܬܐ ܕܗܘܪܦܐ ܠܐ ܡܨܐ ܕܗܘܐ ܠܗ ܢܦܩܐ ܡܢ ܚܕ
ܕܕܓܠܐ.

ܦܫܪܐ

ܡܢܝܢܐ ܕܐܘܪܚܬܐ ܦܪܝܫܬܐ ܠܓܒܝܬܐ ܕܪܥܢܐ = ١٦

ܡܢܝܢܐ ܕܐܘܪܚܬܐ ܦܪܝܫܬܐ ܠܓܒܝܬܐ ܕܚܠܦ ܪܥܢܐ = ١٠

ܡܢܝܢܐ ܕܐܘܪܚܬܐ ܦܪܝܫܬܐ ܠܓܒܝܬܐ ܕܢܛܪ ܐܪܙܐ = ٩

ܚܙܝ ܗܘܝ ܒܬܪܗ ܛܘܦܣܐ ٥ ، ٦ ܠܡܦܩ ܢܝܫܗ ܕܡ ܡܦܩܬܐ (١ . ٢ . ٣) ܒܕ ܠܗ ܛܘܦܣܐ (٥) ܗܘܐ :

ܡܢܝܢܐ ܕܦܠܓܐ = ܡܢܝܢܐ ܕܡܫܚܠܦܢܘܬܐ ܕܟܢܘܫܬܐ ܡܢܝܢܐ ܕܐܣܛܘܟܣܐ
ܝܠܗ (١٥) ܫܩܝܠܐ ܡܢܝܗ (٣) ܒܚܕ ܙܒܢܐ ܘܠܗ ܛܘܦܣܐ (٦) ܒܗܘܐ

ܡܢܝܢܐ ܕܐܘܪܚܬܐ ܦܪܝܫܬܐ = ܡܢܝܢܐ ܕܡܫܚܠܦܢܘܬܐ ܕܟܢܘܫܬܐ ܡܢܝܢܐ
ܕܐܣܛܘܟܣܐ ܝܠܗ (١١) ܫܩܝܠܐ ܡܢܝܗ (٣) ܒܚܕ ܙܒܢܐ .

ܡܦܩܬܐ (١ ـ ٢ ـ ٤)

ܐܢܘ ܗܘܝܐ ܟܠ ܡܢ ، ܪ ∈ ܨ ، ܪ ≤ ، ܒܗܘܝܐ

ܠ (، ܪ) = ܠ ، ܘܡܢܝܢܐ ܡܫܚܠܦܢܘܬܐ ، ܫܩܝܠܬܐ ܪܫ ܪܫ ܘܒܗܘܝܐ

ܠ (، . ܪ) = { ١ ، ܐܢ ܪ = ، ؛ ، (، ـ ١) × ... × (، ـ ܪ + ١) ܐܢ ܪ < ، ؛ ١ ܐܢ ܪ = ܣܦܝܩܐ }

ܘܦܫܩܐ ܝܠܗ ܚܙܝܐ ܕܡܢܝܢܐ ܕܡܫܚܠܦܢܘܬܐ . ܡܢ ܐܣܛܘܟܣܐ ܫܩܝܠܐ
ܪܫ ܪܫ ܒܕ

ܪ < ، ܒܗܐ ܡܢܝܢܐ ܕܐܘܪܚܬܐ ܕܫܩܝܠܝܗ ܒܗ (ܪ) ܡܢ ܐܣܛܘܟܣܐ ܡܢ
ܒܝܢ (،) ܡܢ ܐܣܛܘܟܣܐ ܒܟܠܝܗ ܡܫܪܘܬܐ ܕܗܘܝ.

ܛܘܦܣܐ (٧)

ܐܫܟܚ ܫܘܡܐ ܕܟܠ ܡܢ :

ܐ) ܠ = ܠ (٨ ، ٤) = ٨ × ٧ × ٦ × ٥ = ١٦٨٠

$$\text{ܒ)}\quad \frac{\text{ܠ }(١١ ، ٣)}{\text{ܠ }(١٠ ، ٢)} = \frac{١١ \times ١٠ \times ٩}{١٠ \times ٩} = ١١$$

(ܓ) ܠ (٧ ، ٠) = ١ (ܡܢ ܡܫܘܕܥܬܐ)

(ܕ) ܠ (٥ ، ٥) = ٥! (ܡܢ ܡܫܘܕܥܬܐ)

<u>ܩܐ</u> (٨)

ܐܢ ܗܘܬ ܠ (ܢ ، ٢) = ٤٢ ܐܝܟܢ ܗܘܝܐ ܢ

:

$(٢ . = ܢ (ܢ - ١) = ٤٢$

$ܢ^٢ - ܢ - ٤٢ = ٠$

$(ܢ - ٧)(ܢ + ٦) = ٠$

ܡܢ ܢ = ٧

ܐܘ ܢ = −٦ ܦܝܫ ܡܣܠܝܐ ܒܗܝ ܕܢ ∉ ܛ

<u>ܦܘܫܩܐ</u> (٩) :

ܐܝܟܢ ܗܘܝܐ ܕܟܠ ܡܢ :

(ܐ) ܠ (٨ ، ٥) = ٨ × ٧ × ٦ × ٥ × ٤ = ٦٧٢٠

$$(ܒ) \quad \frac{٨!}{(٨ - ٥)!} = \frac{٨ \times ٧ \times ٦ \times ٥ \times ٤ \times ٣ \times ٢ \times ١}{٣ \times ٢ \times ١}$$

$$= ٨ \times ٧ \times ٦ \times ٥ \times ٤ = ٦٧٢٠$$

ܕܦܠܛܐ ܒܠܚܘܕ ܬܘܒ ܐܢܘ (ܐ) ، (ܒ)

$$ܘܒܝܢ \ ܠ (٨ ، ٥) = \frac{٨!}{(٨ - ٥)!}$$

(ܓ) ܠ (١٥ ، ٧) = ١٥ × ١٤ × ١٣ × ١٢ × ١١ × ١٠ × ٩

$$(ܕ) \quad \frac{١٥!}{(١٥ - ٧)!} = \frac{١ \times ٢ \times ٣ \times ٤ \times ٥ \times ٦ \times ٧ \times ٨ \times ٩ \times ١٠ \times ١١ \times ١٢ \times ١٣ \times ١٤ \times ١٥}{١ \times ٢ \times ٣ \times ٤ \times ٥ \times ٦ \times ٧ \times ٨}$$

$$= ١٥ \times ١٤ \times ١٣ \times ١٢ \times ١١ \times ١٠ \times ٩$$

ܕܦܠܛܐ ܐܢܘ (ܓ) ، (ܕ) ܒܠܚܘܕ ܬܘܒ

ܘܒܝܢ :

$$ܠ (١٥ ، ٧) = \frac{١٥!}{(١٥ - ٧)!}$$

ܘܐܦ ܕܐܗܐ ܡܫܪܪܬܐ ܝܠܗ ܠܐܡܪܐ ܒܪܘܪܬܐ ܦܘܫܚܬܐ :

$$\text{ܠ}(\text{ܢ} ، \text{ܪ}) = \frac{\text{ܢ}!}{(\text{ܢ} - \text{ܪ})!}$$

ܦܘܫܩܬܐ (١٠) :

ܐ، ܐܢ ܗܘܐ ܠ (٦ ، ٣) = ܠ (٦ ، ܪ) ܡܢܐ ܝܠܗ ܩܝܡܬܐ ܕ ܪ

ܦܘܢܝܐ :

$$\text{ܠ}(٦ ، ٣) = \frac{٦!}{(٦ - ٣)!} = \frac{٦!}{٣!}$$

$$\text{ܠ}(٦ ، ٣) = \frac{٦!}{(٦ - \text{ܪ})!}$$

$$\frac{٦!}{٣!} = \frac{٦!}{(٦ - \text{ܪ})!} \Leftarrow ٣! = (٦ ، \text{ܪ})!$$

$$٣ = ٦ - \text{ܪ} \Leftarrow \text{ܪ} = ٣$$

ܘܡܫܪܪܬܐ ܝܠܗ ܠܐܡܪܐ ܒܪܘܪܬܐ ܦܘܫܚܬܐ ܕ:

ܐ، ܐܢ ܗܘܐ ܠ (ܢ ، ܪ) = ܠ (ܢ ، ܗ) ܒܗܝ ܕܪܐ ܪ = ܗ

ܦܘܫܩܬܐ (١١)

ܐ) ܡܢܝܢܐ ܕܡܢܝܢܐ ܡܬܘܪܟܐ ܕܡܨܐ ܕܒܠܝܢ ܡܢܝܢ ܡܢ ܬܪܝܢ ܕܡܢܐ ܦܪܝܫܐ
ܥܡܝܠܐ ܡܢ (٦) ܕܡܢܐ ܕܠܐ ܬܢܝܬܐ ܕܕܡܢܐ ܚܕܐ :
ܡܢܝܢܐ ܕܡܨܝܠܦܢܘܬܐ ܕܐܪܬܐ ܕܡܢܐ ܥܡܝܠܐ ܬܪܝܢ ܬܪܝܢ ܘܗܘܝܐ
ܠ (٦ ، ٢) = ٦ × ٥ = ٣٠ ܡܢܝܢܐ

ܒ) ܡܢܝܢܐ ܕܡܢܝܢܐ ܡܬܘܪܟܐ ܕܡܨܐ ܕܒܠܝܢ ܡܢܝܢ ܡܢ ܬܠܬܐ ܕܡܢܐ ܦܪܝܫܐ
ܥܡܝܠܐ ܡܢ (٦) ܕܡܢܐ ܕܠܐ ܬܢܝܬܐ ܕܕܡܢܐ ܚܕܐ ܡܢܝܢܐ ܕܡܨܝܠܦܢܘܬܐ
(٦) ܕܡܢܐ ܬܠܬܐ ܬܠܬܐ ܘܗܘܝܐ ܠ (٦ ، ٣) = ٦ × ٥ × ٤ = ١٢٠
ܡܢܝܢܐ

ܓ) ܘܒܐܝܟܢܘܬܐ ܕܡܠܬܐ ܐܪܒܥܐ ܕܡܢܐ ܕܠܐ ܬܢܝܬܐ ܒܗܘܐ
ܡܢܝܢܐ ܕܡܢܝܢܐ = ܠ (٦ ، ٤)
= ٦ × ٥ × ٤ × ٣ = ٣٦٠ ܡܢܝܢܐ

ܕܘܪ̈ܫܐ (١ - ٢)

ܚܫܘܒ ܬܘܡܘܬܐ ܕܟܠ ܚܕ ܡܢ ܗ̈ܢܐ ܕܐܬܝܐ:

(ܐ) ܠ(١٠، ٥) (ܒ) ܠ(٩، ٤)

(ܓ) ܠ(٧، ٦) / ܠ(٦، ٥) (ܕ) ܠ(٦، ٤) / ٥!

ܐܫܟܚ ܬܘܡܘܬܐ ܢ، ܒܟܠ ܚܕ ܡܢ ܗ̈ܢܐ ܕܐܬܝܐ

(ܐ) ܠ(ܢ، ٢) = ٧٢ (ܒ) ܠ(ܢ، ٥) = ٨ × ܠ(ܢ، ٤)

ܟܡܐ ܡܢܝܢܘܬܐ ܦܪ̈ܝܫܬܐ ܡܨܝܢܢ ܠܗ ܠܡܥܒܕ ܒܐܝܕ ܒܬܫܠܡܬܐ ܕܐܬܘ̈ܬܐ
ܐܬܘܬܐ ܦܪ̈ܝܫܬܐ ܡܢ ܒܝܬ ܐܬܘ̈ܬܐ ܕܝܘܢ̈ܝܐ (ܝܘܢܝܐ).

ܐܢ ܒܝܬ ܐܬܘܬܐ ܗ = {١، ٢، ٣، ٤، ٥، ٦، ٧} ܟܡܐ ܡܢܝܢ̈ܐ ܡܨܝܢܢ ܠܗ
ܠܡܥܒܕ ܕܠܐ ܡܫܘܚܠܦܘܬܐ ܕܡܢܝܢܐ ܐܢ ܡܢܝܢ̈ܐ ܒܗܘܢ ܕܡܢܝܢ ܡܢ ܡܢܘܬܐ
ܠܗ ܡܢ:

(ܐ) ܚܕ ܡܢܝܢܐ (ܒ) ܬܪܝܢ ܡܢܝܢܐ (ܓ) ܬܠܬܐ ܡܢܝܢܐ (ܕ) ܐܪܒܥܐ ܡܢܝܢܐ

(ܗ) ܚܡܫܐ ܡܢܝܢܐ (ܘ) ܐܫܬܐ ܡܢܝܢܐ (ܙ) ܫܒܥܐ ܡܢܝܢܐ

ܚܕ ܡܕܪܫܐ ܐܝܬ ܠܗ ܫܘܩܐ ܥܠ ܛܘܪܐ ܕܪܟܐ ܚܕ ܡܢ ܡܕ̈ܝܢܬܐ ܕܡܕܪܫܐ
ܘܠܝܬ ܕܪ̈ܓܐ ܘܠܐ ܕܪ̈ܓܐ ܘܬܪܥܐ ܐܚܪܝܐ. ܟܡܐ ܐܝܬ ܠܗ ܡܢܝܢ̈ܐ ܕܦܬܓܡܐ ܕܦܬܓܡܐ
ܡܢ ܫܘܩܐ ܐ، ܦܬܓܡܐ ܢܦܩܬܐ ܕܡܢܝܢ̈ܐ ܕܠܗܘܢ ܠܗ ܫܘܩܐ ܐܝܬ ܠܗ (٢٠٠)
ܦܬܓܡܐ.

ܙܘܘܓܐ :

ܡܬܠܐ (١٢) :

ܐܢ ܗܘܐ ܠܢ ܣܘܕܪܐ ܣ = {١ ، ٢ ، ٣}

ܟܠܗܘܢ ܕܣܘܕܪ̈ܐ ܡܢܬ̈ܝܢܝܬܐ ܡܢ ܣ ܘܡܫܘܚܬܗܝܢ ܡܢ ܬܪܝܢ ܗܡܢܘܢ ܐܢܘܢ

{١ ، ٢} ، {١ ، ٣} ، {٢ ، ٣} .

ܚܙܝ ܗܘܐ ܒܗܢܐ ܡܬܠܐ ܠܐ ܡܫܬܟܚ ܫܘܚܠܦܐ ܠܣܕܪܘܬܐ ܕܡܬܠܬܐ ܠܘ ܟܠ ܡܬܠܐ ܥܡ ܡܬܠܐ {١ ، ٢} ܐܝܟ ܕܝܠܗ {٢ ، ١}

ܘܡܬܠܐ {١ ، ٣} ܐܝܟ ܕܝܠܗ

{٣ ، ١} ܘܡܬܠܐ {٢ ، ٣} ܐܝܟ ܕܝܠܗ {٣ ، ٢}.

ܐܝܟ ܕܗܘܐ ܡܬܠܐ ܘܐܝܟ ܡܢܝܢܐ ܕܬܪܝܢ ܡܬܠܬܐ ܡܢ ܟܠ ܬܠܬܐ ܡܬܠܬܐ ܕܠܐ ܫܘܚܠܦܐ ܠܣܕܪܘܬܐ ܕܡܬܠܬܐ ܕܟܠܗܝܢ ܡܬܠܬܐ ܦܫܝܩ ܡܢܝܢ ܙܘܘܓܐ ܘܗܢܐ ܡܬܠܐ ܦܫܝܩ ܡܢܝܢ ܙܘܘܓܐ (٣) ܣܡܝܟܐ ܡܢ ܬܪܝܢ ܬܪܝܢ.

ܦܢܕܐ (١ - ٣ - ١) :

(١) ܙܘܘܓܐ ܕܣܘܕܪܐ ܡܬܚܡܬܐ ܡܢ ܡܬܠܬܐ ܐܝܟ ܡܬܠܐ ܕܣܕܝܪܐ ܐܘ ܠܟܠܗܝܢ ܡܬܠܬܐ ܕܠܐ ܫܘܚܠܦܐ ܠܣܕܪܘܬܐ ܕܡܬܠܬܐ ܒܗ ܐܝܟ ܡܬܠܬܐ.

(٢) ܡܢܝܢܐ ܕܙܘܘܓܐ . ܡܢ ܡܬܠܬܐ ܣܡܝܟܐ ܪ ܪ ܟܕ (ܪ ≤ ܢ)

(٣) ܐܝܟ ܡܢܝܢܐ ܕܙܘܘܓܐ ܕܡܬܚܡܐ ܪ ܡܢ ܡܬܠܬܐ ܕܠܐ ܫܘܚܠܦܐ ܠܣܕܪܘܬܐ ܕܐܝܟ ܡܬܠܬܐ ܘܦܫܝܩ ܪܡܙܐ ܠܗܘܐ ܒܪܡܙܐ ܣ ܐܘ ܒܪܡܙܐ (ܪ ̈) ܐܘ ܣ (ܢ ، ܪ)

(٣) ܣ = (ܪ ̈) = $\dfrac{ܠ\,(ܢ\,،\,ܪ)}{ܪ\,!}$

(٤) ܣ (ܢ ، ܢ) = ܠ (ܢ ، ܢ) = ١

ܦܬܐ (١٣)

ܐܫܟܚ ܡܫܘܚܬܐ :

$$(\text{ܐ})\ \binom{٥}{٣} = \frac{\text{ܠ}(٥ ، ٣)}{٣!} = \frac{٥ \times ٤ \times ٣}{٣ \times ٢ \times ١} = ١٠$$

$$(\text{ܒ})\ \text{ܩ}^{١٠}_{٤} = \frac{\text{ܠ}(١٠ ، ٤)}{٤!} = \frac{١٠ \times ٩ \times ٨ \times ٧}{٤ \times ٣ \times ٢ \times ١} = ٢١٠$$

$$(\text{ܓ})\ \text{ܩ}(١٥ ، ١٢) = \frac{\text{ܠ}(١٥ ، ١٢)}{١٢!}$$

$$= \frac{١٥\times١٤\times١٣\times١٢\times١١\times١٠\times٩\times٨\times٧\times٦\times٥\times٤}{١٢\times١١\times١٠\times٩\times٨\times٧\times٦\times٥\times٤}$$

$$= \frac{١٥ \times ١٤ \times ١٣}{٣ \times ٢ \times ١} = ٤٥٥$$

$$(\text{ܕ})\ \text{ܩ}(١٥ ، ٣) = \frac{١٥ \times ١٤ \times ١٣}{٣ \times ٢ \times ١} = ٤٥٥$$

ܚܙܝܐ ܗܘܐ ܡܢ (ܓ) ، (ܕ) ܩ(١٥ ، ١٢) ܩ(١٥ ، ٣)

ܘܚܘܕܬܐ ܓܘܢܝܬܐ ܡܢܝܢܬܐ ܗܠܝܢ ܠܐܡܪܐ
$\text{ܩ}(\text{ܢ} ، \text{ܪ}) = \text{ܩ}(\text{ܢ} ، \text{ܢ} - \text{ܪ})$

ܦܬܐ (١٤)

ܟܡܐ ܡܬܠܬܐ ܫܘܝܐ ܡܨܝܢܢ ܠܗ ܕܢܒܢܝܢ ܒܢܘܩܙܐ ܡܢ ܒܢܘܩܙܐ ܕܡܢܝܢܐ ܕܡܬܠܬܐ (٦) ܠܐ ܢܦܠܝܢ ܫܠܫܐ ܡܢܗܘܢ ܥܠ ܚܕ ܫܘܝܘܬܐ .

ܐ :

$$\text{ܡܢܝܢܐ ܕܡܬܠܬܐ ܫܘܝܐ} = \text{ܩ}^{٦}_{٢} = \text{ܩ}(٦ ، ٢) = \frac{٦ \times ٥}{٢ \times ١} = ١٥\ \text{ܡܬܠܬܐ.}$$

ܦܬܐ (١٥) :

ܟܡܐ ܡܬܠܬܐ ܡܨܝܢܢ ܠܗ ܡܢܝܬܐ ܥܠܝܗ ܡܢ ܒܢܘܩܙܐ ܡܢ ܢܘܩܙܐ ܡܢܝܢܐ ܕܡܬܠܬܐ (١٢) ܢܘܩܙܐ ܫܠܫܐ ܡܢܗܘܢ ܠܐ ܢܦܠܝܢ ܥܠ ܚܕ ܫܘܝܘܬܐ.

ܡܢܝܢܐ ܕܩܘܛܠܐ = $^{١٢}ܩ_{٣}$ = ܩ(١٢ ، ٣)

$= \dfrac{١٢ \times ١١ \times ١٠}{٣ \times ٢ \times ١} = ٢٢٠$ ܩܘܛܠܐ

ܕܘܪܫܐ (١ - ٣)

ܫ١ : ܐܫܟܚ ܫܘܘܝܘܬܐ ܕܟܠ ܚܕ :

(ܐ) ܩ(٦ ، ٢)   (ܒ) $^{١٠}ܩ_{ܢ}$ (ܓ) $\binom{١٢}{٨}$ (ܕ) $\binom{١٥}{١٥}$

(ܗ) $^{ܢ}ܩ_{ܢ}$.   (ܘ) $\binom{ܢ}{١}$ (ܙ) ܠ(ܢ ، ܢ)   (ܚ) ܩ(ܢ ، ܢ)

(ܛ) ܠ(ܢ ، ١)   (ܝ) $\binom{ܢ}{١}$ (ܟ) ܠ(ܢ ، ٠)   (ܠ) ܩ(ܢ ، ٠)

ܫ٢ : ܐܫܟܚ ܫܘܘܝܘܬܐ ܕܢ ܗܘܐ :

(ܐ) $\binom{ܢ}{٢} = ١٠$ (ܒ) $\binom{ܢ+١}{٣} = ٢\binom{ܢ}{٢}$

ܫ٣ : ܐܝܟܢܐ ܡܢ ܡܢܝܢܐ ܕܐܝܬ ܠܗ ܫܘܝܐ ܘܐܝܟܢܐ ܦܘܪܫܐ :

(ܐ) $\binom{١٠}{٦} = \binom{١٠}{٤}$

(ܒ) $\binom{٢٥}{٢٣} = \dfrac{ܠ(٢٥ ، ٢)}{٢!}$

(ܓ) ܐܢ ܗܘܐ $\binom{ܢ}{٦} = \binom{ܢ}{٤}$ ܒܗܘܢ ܢ = ١٠

(ܕ) ܡܢܝܢܐ ܕܩܘܛܠܐ ܡܫܘܚܬܐ ܕܢܬܬܒܢܘܢ ܥܠ ܬܠܬܐ ܢܘܩܙܐ
ܕܠܐ ܢܗܘܘܢ ܥܠ ܣܪܛܐ ܚܕ ܡܢ ܥܣܪܐ ܢܘܩܙܐ ܕܐܝܬ
(١٠) = ܩ(١٠ ، ٣)

(ܗ) ܡܢܝܢܐ ܦܘܪܫܐ ܠܐ ܫܘܝܐ ܦܘܪܫܐ، ܡܢܝܢܐ ܕܐܘܪܚܬܐ ܕܚܫܒܬܐ

ܕܬܠܬܐ ܡܢܝܢܗ = ܠ(٧ ، ٣)

(ܗ) ܡܢܝܢܐ ܕܐܘܪܚܬܐ ܕܫܟܝܚܬܐ ܕܒܝܢ ܦܪܘܦܐ ܡܢ (٦) ܦܪܘܦܐ ܕܠܐ ܚܕܐ ܕܗܕܝܪܘܬܐ ܠܗ ܫܟܝܚܬܐ = ١٥ ܐܘܪܚܬܐ.

(ܘ) ܠ (٣ ، ٠) − ٢ = |٠ = − ١

(ܙ) ܠܟܠ ܢ ، ܪ ∈ ܨ ، ܐܝܬ ܠ (ܢ ، ܪ) = ܠ(ܢ ، ܢ) ܒܐܝܬܝܐ ، ܪ = ܢ

ܐ. ܚܒܪܢܐ ܦܪܫ ܕܒܘܚܪܢܐ ܕܡܢܝܢܬܐ ܒܝܘܠܦܢܐ ܕܝܘܡܐ ܟܠ ܐܝܟܐ ܒܝܐܠܝܢ ܦܪܝܫܐ. ܒܟܠ ܐܘܪܚܬܐ ܡܢ ܐ ܠܗܘܢ ܕܒܘܚܪܢܐ ܦܘܫܐ ܟܠ ܚܕܐ ܡܢܝܢܗ.

ܡܠܒܫ ܠܦܘܬܐ ܗܕܝܐ ܠܗ ܟܠ ܡܢ ܗܢܐ ܕܐܬܐ:

(ܐ) ܡܢܝܢܐ ܕܐܘܪܚܬܐ ܕܫܟܝܚܬܐ ܕܡܒܠܬܐ ܗܠܝܬܐ ܡܢ ܒܝܢ (١٠) ܦܪܘܦܐ ܒܗܘܐ

ܐ) ܠ (١٠ ، ٣) (ܒ) ($\frac{١٠}{٣}$) (ܓ) $\frac{|١٠}{|٣}$

ܕ) ܐܦܢܕ ܡܢ ܗܢܐ ܠܝܕܥܬܐ

ܒ. ܐܝܬ ܡܢܝܢܐ ܕܒܘܢܝܐ ܡܬܚܠܦܬܐ ܗܕܝܬܐ ܕܡܢܝܢܬܐ ܚܕܐ ܡܠܬܐ ܡܢ ܒܘܢܝܐ ܡܢܝܢܐ ܕܐܣܛܘܟܣܐ ܒܗܘ (٦) ، ܒܟܘܢܐ.

(١) ١٥ (٢) ٦ (٣) ٤ (٤) ٢

ܓ. ܡܢܝܢܐ ܕܡܫܠܡܢܐ ܗܕܝܐ ܕܡܢܝܢܬܐ ܚܕܐ ܕܡܫܠܡ ܒܝܢ ܗܕܐ ܕܪܐ ܡܢ ܕܪܐ ܕܡܛܠܠܐ ܒܬܪܬܐ ܒܗܘ

(١) ٦ × ٦ (٢) ܩ (٦ ، ٢)

(٣) ܠ (٦ ، ٢) (٤) |٦

$$\left(\frac{٦٨}{٦٠}\right) \div \left(\frac{٦٨}{٨}\right) =$$

(١) ٦٨ (٢) $\frac{٨}{٦٠}$ (٣) ١ (٤) $\frac{ܠ (٦٨ ، ٨)}{٨}$

ܕ. ܐܝܬ ܩ = {١ ، ٢ ، ٣ ، ٤ ، ٥ ، ٦ ، ٧ ، ٨ ، ٩}

ܡܢܝܢܐ ܕܡܢܝܢܐ ܕܪܫܐ ܕܟܠܝܢ ܡܢܝܢܐ ܡܬܒܪܝܐ ܡܢ ܐܪܒܥܐ ܦܪܝܫܐ ܦܪܝܫܐ ܘܡܢܝܢܬܐ ܚܕܐ ܡܬܕܥܝܢ ܡܢ ܐܣܛܘܟܣܐ ܩ ܗܘ:

(١) |٩ (٢) ($\frac{٩}{٤}$)

(٣) |٤ (٤) ܠ (٣ ، ٢) (٥) ܐܦܢܕ ܡܢ ܗܢܐ ܠܝܕܥܬܐ

٥) ܐ. ܗ = {٢ ، ١ ، ٠} ، ܙ = {ܐ ، ܒ ، ܓ} ܡܢܝܢܐ ܕܡܦܠܚܢܘܬܐ ܡܚܫܠܦܬܐ ܡܢ
ܗ ← ܙ ܗܝܠܗ :

١) [٢]   ٢) ٥!

٣) ܗ (٣ ، ٢)   ٤) ܠ (٣ ، ٢)

٦) ܐ. ܢܗܘܐ ܠ (ܢ ، ٢) = ٩٠ ܒܗܘܝܐ ܢ =

١) ١٠   ٢) ٢٤

٣) ٤٥   ٤) ١٨

ܒ: ܒܟܡܐ ܐܘܪܚܬܐ ܡܨܝܢܐ ܠܗ ܓܒܪܐ ܕܐܙܠ ܡܢ ܡܕܝܢܬܐ ܠܡܕܝܢܬܐ ܡܢ (٥) ܫܠܘܦܝܢܬܐ. (٧) ܫܠܘܦܐ ܡܢ ܒܝܬ ܒܢܘܬܐ ܡܕܝܢܬܐ ܡܢ (٢٠) ܫܠܘܦܐ. (٣٠) ܫܠܘܦܐ.

ܦܣܘܩܐ ܬܪܝܢܐ

ܢܒܥܐ ܕܐܬܐ

(٢ - ١) ܠܘܬܪܐ :

ܣܕܝܩܠܗ ܘܕܪܫܠܝ ܗܘܟܠܐ ܕܐܬܐ ܘܐܢܫܝܘܬܐ ܕܡܫܟܚܬܐ
ܣܘܟܢܐ ܕܐܬܐ ܓܢܒ ܥܠ ܗܓܠܘܬܐ ܡܢܟܝܢ ܠܟܠܗ ܘܦܠܚܢܐ ܕܣܘܕܥܐ
ܘܡܫܟܚܬܐ ܕܚܫܒܢܐ ܒܝ ܕܐܝܟ ܠܡܢܝܢܐ ܕܐܬܐ ܐܝܟ ܕܐܬܪܐ ܕܐܬܐ
ܐܬܐ.

ܘܡܬܚܫܒܬܐ ܗܠܗ ܠܐܝܟ ܘܕܘܕܐ ܣܘܟܠܐ ܡܬܠܬܐ ܠܐ ܡܬܡܢܝܢ ܠܗ ܒ ܒܝ
ܒܘܟܬܐ ܕܡܢܝܢܐ ܒܪܝܬܐ ܘܗܠܗ :

ܚܠ ܐ ∋ ܒ ܒܗܘܐ $\frac{2}{ܡܢܝܢܐ}$ ܠܐ ܡܬܡܢܝܢܬܐ ܠܗ ܒ ܐܝܟ

$\frac{1}{ܡܢܝܢܐ}$ ، $\frac{-3}{ܡܢܝܢܐ}$ ، $\frac{ܡܢܝܢܐ}{ܡܢܝܢܐ}$ .... ܘܗܕܐ

ܡܢܝܢܐ)^{ܡܢܝܢܐ} ܠܐ ܡܬܡܢܝܢܬܐ ܒ

(٢ - ٢) ܢܒܥܐ ܕܐܬܐ ܓܢܒ ܒܡܘܬܐ

ܗܘܟܠܐ ܕܢܒܥܐ ܡܢ ܡܫܬܟܠܢܘܬܐ ܐܫܬܐ ܓܘ
ܣܘܟܐ ܒܝ ܐܟܠ ܗܠܗ ܐܝܟ ܡܢ ܡܫܬܟܠܢܘܬܐ ܐܢܫܝܬܐ ܐܚܪܢܐ ܐܝܟ
ܡܫܬܟܠܢܘܬܐ ܕܐܡܝܢܘܬܐ ܘܡܬܠܬܐ ܘܠܡܬܪܘܬܐ ܘܡܕܪܫ ܐܝܟ ܕܢܒܥܐ
ܐܘܪܚܐ ܦܫܝܛܬܐ ܒܐܘܪܚܐ ܕܠܗܘܝܐ ܕܠܐ ܡܫܬܕܪܐ ܠܗ ܐܪܒܥܬܐ
ܐܘܪܚܐ ܕܡܢܝܢܐ ܐܝܟ ܡܬܕܪܬܐ ܕܢܒܥܐ ܒܝܘܡܬܐ ܕܡܢܝܢܬܐ ܘܡܗܘܐ
ܣܘܡܐ. ܕܠܐܝܟ ܒܐܬܐ ܡܬܐܘܪܚܐ ܕܗܘܐ ܦܫܝܛܬܐ ܟܠܢ ܡܫܟܚܬܐ
ܢܒܥܐ ܕܐܬܐ ܓܢܒ ܒܡܘܬܐ ܡܬܚܙܝܬܐ .

ܡܢܝܢܐ (١)

ܒ ܗܘܐ ܕ(ܗ) = $\frac{ܗ^{2} - 1}{ܗ - 1}$ ܐܬܐ

ܠܟܬܐ ܗܠܗ ܕܠܝܬܠܐ ܒܝ ܕܐܝܟ ܠܗܘܐ ܐܬܐ ܗܠܗ ܒ |١| ܗܘܐ ܘܕܡ ܐܬܐ
ܐ ܗܠܗ ܡܬܡܢܝܢܬܐ ܓܢܒ ܗ = ١ ، ܗܘܐ ܘܕܡ ܕ(ܗ) ܠܐ ܗܠܗ ܡܬܡܢܝܢܬܐ

ܐܢܟܝ ܣ←١ ܘܦܘܢܩܝܬܐ ܕܝܕܝܥܬܐ ܠܗܘ ܐܝܬ ܐܝܬܐ ܠܗ ܒܐܘܚܡܐ ܥܒܪܐ
ܐܘܚܡܐ (٢ ـ ١)

ܐܘܚܡܐ (٢ ـ ١)

ܘܟܕ ܚܙܝܢ ܠܦܘܢܩܝܬܐ ܕ(ܣ) ܐܢܟܝ ܣ ܡܩܪܒܐ ܩܠܝܠ ܘܩܠܝܠ (ܐܘ ܦܝܫܐ ܐܣܝܪܬܐ ܕܒܝܫ ܪܒܐ) ܡܢ ܡܢܝܢܐ (١) (ܒܐܡܪܝܢ ܣ ܢܩܘܒ ܡܢܝܚܠܦܐ ܡܩܪܒܐ ܠܦܘܢܩܝܬܐ ܕܐܝܬ) ܡܩܪܒܐ ܪܒܐ ܪܒܐ ܡܢ ܡܢܝܢܐ (١) ܘܐܝܬܐ ܫܘܟܠܗ ܢܩܘܒ ܣ ≠ ١ ܘ ܣ ܡܩܪܒܐ ܡܢ (١) ܡܢ ܣܛܪܐ ܗܘܐ ܕܝܢ ܣ > ١ ܘܦܝܫ ܪܫܝܡܐ ܠܗܘܐ ܣ ← ١. ܐܘ ܣ ܡܩܪܒܐ ܡܢ ܣܡܠܐ ܡܢ (١) ܗܘܐ ܕܝܢ ܣ < ١ ܘܦܝܫ ܪܫܝܡܐ ܠܗܘܐ ܣ ← ١_ ܘܣܘܪܓܕܐ ܕܐܬܐ ܒܬܪܗ ܠܗܘ ܐܝܬ ܡܩܪܒܘܬܐ .

ܣ	١,١	١,٠١	١,٠٠١	١,٠٠٠١	١,٠٠٠٠١	←	١	→	٠,٩٩٩٩	٠,٩٩٩	٠,٩٩	٠,٩
ܕ(ܣ)	٢,١	٢,٠١	٢,٠٠١	٢,٠٠٠١	٢,٠٠٠٠١	← ٢	(٢)	٢ →	١,٩٩٩٩	١,٩٩٩	١,٩٩	١,٩

ܚܙܝܢ ܡܢ ܕܐܝܬ ܣܘܪܓܕܐ ܕܐܢܟܝ ܣ ܩܪܒܐ ܦܘܢܩܝܬܐ ܡܢ ܥܠܝܬܐ ܕܡܢܝܢܐ < ١,١ ، ١,٠١ ، ١,٠٠١ ، ١,٠٠٠١ ، ... > ܣ ܒܩܪܒܐ ܡܩܪܒܐ ܩܠܝܠ ܘܩܠܝܠ ܡܢ ܡܢܝܢܐ (١) ܒܕ ܣ ١ ܗܝܕܝܢ ܒܐܡܪܝܢ ܣ ← ١ ܘܥܠܝܬܐ ܕܡܢܝܢܐ ܠܦܘܢܩܝܬܐ ܕ(ܣ) ܐܬܩܪܒ ܠܦܘܢܩܝܬܐ ܣ ܢܩܘܒ < ٢,١ ، ٢,٠١ ، ٢,٠٠١ ، ٢,٠٠٠١ .... > ܩܪܒܐ ܡܩܪܒܬܐ ܩܠܝܠ ܘܩܠܝܠ ܡܢ ܡܢܝܢܐ (٢) ܗܝܕܝܢ ܒܐܡܪܝܢ ܕ(ܣ) ← ٢. ܘܗܝܕܝܢ ܦܝܫܐ ܐܣܝܪܬܐ ܕܒܝܫ ܡܢ ܣܛܪܐ ܝܡܝܢܐ ܕ ܐܢܟܝ ܣ ܡܩܪܒܐ ܡܢ (١) ܗܝ (٢) ܘܦܝܫܐ ܟܬܝܒܬܐ ܐܝܟ ܕܐܬܐ :

$$\lim_{ܣ \to ١^{+}} ܕ(ܣ) = \lim_{ܣ \to ١^{+}} \frac{ܣ^{٢} - ١}{ܣ - ١} = ٢^{+}$$

ܘܐܦ ܚܙܝܢ ܗܘܐ ܡܢ ܕܐܝܬ ܣܘܪܓܕܐ ܕܐܢܟܝ ܩܪܒܐ ܣ ܥܠܝܬܐ ܡܢ ܥܠܝܬܐ ܕܡܢܝܢܐ < ٠,٩ ، ٠,٩٩ ، ٠,٩٩٩ ، ٠,٩٩٩٩ ، ... > ܣ ܒܩܪܒܐ ܡܩܪܒܐ ܩܠܝܠ ܘܩܠܝܠ ܡܢ ܡܢܝܢܐ (١) ܘ ܣ < ١ ܗܝܕܝܢ ܐܡܪܝܢ ܣ ← ١⁻ ܘܥܠܝܬܐ ܕܡܢܝܢܐ

...ܣܘܡܟܐ ܕ(ܣ) ܐܢܬܘܢܝܐ ܠܣܘܡܟܐ ܣ ܡܠܝܐ < ١,٩ ، ١,٩٩ ، ١,٩٩٩ ، ١,٩٩٩٩ ،
١,٩ ، ...> ܒܕܐ ܡܩܪܒܐ ܢܛܝܐ ܘܢܛܝܐ ܡܢ ܡܢܝܢܐ (٢) ܒܐܡܪܝܢ ܕ(ܣ) ← ٢ –
...ܡܢ ܦܢܝܐ ܐܡܝܢܐ ܢܦܠܐ ܡܢ ܣܡܠܐ ܠܐܢܐ ܣ ܡܩܪܒܐ ܡܢ (١) ܒܡܐ (٢)
...ܕܐ ܟܬܝܒܐ ܒܐܣܟܡܐ .

ܠܚܡ ܕ (ܣ) = ܠܚܡ (ܣ٢ – ١)/(ܣ – ١) = ٢-
(ܣ ← ١-) (ܣ ← ١-)

...ܡܢ ܦܢܝܐ ܐܡܝܢܐ ܠܐܢܐ ܕ ܢܦܠܐ ܐܝܟ ܣ ܡܩܪܒܐ ܡܢ (١) ܘܐܢܐ ܢܦܠܐ ܒܡܐ
ܘܦܝܫ ܟܬܝܒܐ ܒܐܣܟܡܐ .

ܠܚܡ ܕ (ܣ) = ܠܚܡ (ܣ٢ – ١)/(ܣ – ١) = ٢
(ܣ ← ١) (ܣ ← ١)

...ܦܬܐ (٢)

ܒܚܘܢ ܗܘܐ ܕ(ܣ) = ٢ ܣ + ٣

...ܠܚܡܐ ܝܠܗ ܕܠܚܡܐ ܒܕ ܕܘܟܐ ܠܐܗܐ ܐܝܬ ܝܠܗ (ܒ) ܗܢܐ ܘܕܡ ܕ(ܣ) ܝܠܗ
...ܕܐ ܠܟܠ ܣ (ܒ) ܘܒܚܘܢ ܚܘܣ ܒܬܘܡܟܐ ܕܗܘܐ ܐܗܐ ܐܝܟ ܣ ܡܩܪܒܐ
...ܘܢܛܝܐ ܡܢ ܡܢܝܢܐ (٢) ܗܢܐ ܘܕܡ ܣ = ٢ . ܣ ܡܩܪܒܐ ܡܢ ٢ ܡܢ ܡܢܝܢܐ ܗܢܐ
...ܡ ܣ >٢ ، ܣ ܡܩܪܒܐ ܡܢ (٢) ܡܢ ܣܡܠܐ ܗܢܐ ܘܕܡ ܣ < ܠܚܡܐ ܒ)
...ܘܗܘܝܐ ܕܐܝܬ ܠܚܡܐ ܝܠܗ ܠܐܗܐ ܡܩܪܒܬܐ .

١,٩	١,٩٩	١,٩٩٩	١,٩٩٩٩	→	٢	←	٢,٠٠٠١	٢,٠٠١	٢,٠١	٢,١
٦,٨	٦,٩٨	٦,٩٩٨	٦,٩٩٩٨	→٧	⑦	٧ ←	٧,٠٠٠٢	٧,٠٠٢	٧,٠٢	٧,٢

...ܐ ܗܘܚ ܕܐܝܟ

← ٢، ܒܗܘܐ ܕ (ܣ) ← ٧، ܗܢܘܕܡ ܠܚܡ ܕ (ܣ) = ٧،
(ܣ ← ٢،)

← ٢- ܒܗܘܐ ܕ (ܣ) ← ٧- ܗܢܘܕܡ ܠܚܡ ܕ (ܣ) = ٧
(ܣ ← ٢)

...ܕܗܘ ܘܡܢ ܠܗ ܦܘܬܐ (١) ܦܝܫ ܕܐܡܪܝܢ :

ܠܚܡ ܕ (ܣ) = ܠܚܡ (٢ ܣ + ٣) = ٧
(ܣ ← ٢) (ܣ ← ٢)

ܡܫܘܕܥܢܘܬܐ :

ܬܠܘܝܬ ܐ، ܒܪܐܠܗ ܐܝܟ ܕܪܗܐ ܘܦܩܬܐ ܦܬܝܚܬܐ ܒܐܚܕܐ ܦܘܫܩܐ ܦܬܝܚܬܐ ܗܢܐ ܘܕܝܢ :

ܐ. ܚܘܫܒ (ܕ) ܐܝܬܝܗ ܘܡܛܝܐ ܣ ← ܐ (ܣܕ ܐ ܡܢܝܢܐ ܬܚܘܡܐ ܡܫܚܠܦܢܐ ܒܥܝܢܐ ܕܡܬܚܘܝܐ ܐܘ ܠܐ ܡܬܚܘܝܐ ܠܬܚܘܡܐ ܕܐܝܬܐ . ܣ ≠ ܐ ܘ ܣ ← ܐ. ܗܢܐ ܘܕܝܢ ܣ ܡܩܪܒܐ ܡܢ ܡܢܝܢܐ ܡܢ ܐ ܣܕ ܣ > ܐ ܐܘ ܣ ← ܐ- ܗܢܐ ܘܕܝܢ ܣ ܡܩܪܒܐ ܡܢ ܣܡܠܐ ܡܢ ܐ ܣܕ ܣ < ܐ ܘܐܗܐ ܣܘܟܠܐ ܝܠܗ ܕܩܘܪܒܐ ܒܝܢ ܣ . ܐ ܐܝܟ ܡܛܝܐ ܘܡܛܝܐ) ܡܬܒܥܐ ܐܗܐ ܕ(ܣ) ← ܠ ܣܕ ܠ ܡܢܝܢܐ ܬܚܘܡܐ ܠܣܘܦܐ ܗܢܐ ܘܕܝܢ ܩܘܪܒܐ ܒܝܢ ܕ(ܣ) ، ܠ ܐܝܟ ܡܛܝܐ ܘܡܛܝܐ ܘܗܘܝܢ ܦܬܐ ܐܣܝܪܬܐ

$$\lim_{ܣ \to ܐ} ܕ(ܣ) = ܠ$$

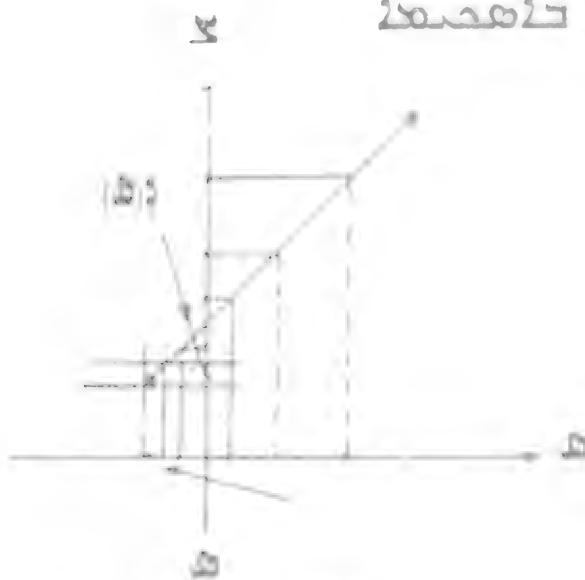
ܘܐܗܐ ܠܐ ܝܠܗ ܡܫܘܕܥܬܐ ܠܣܟܐ ܕܐܝܬܐ ܐܝܟ ܚܘܫܒܐ ܐܠܐ ܐܠܘܬܐ ܦܫܝܛܬܐ ܠܣܘܟܠܐ ܕܣܟܐ

ܘܠܐ ܝܠܗ ܒܕܓܐ ܕܗܘܢܐ ܐܝܬܐ ܡܫܘܕܥܬܐ ܠܣܟܐ ܘܣܘܟܠܐ ܕܡܢܝܢܐ ܐܠܝܒܘܬܐ ܡܥܒܕܢܘܬܐ ܕܡܢܝܢܐ ܐܚܪܢܐ ܘܡܒܥܝܢܐ ܕܗܘܢܐ ܐܝܬܐ ܡܫܘܕܥܬܐ ܠܣܟܐ ܬܠܘܝܬ ܐܘ ܠܣܘܟܠܐ ܬܠܘܝܬ ܐܝܟ ܡܬܠܘܦܐ ܕܐܬܐ :

ܕ : { ܣ : ܣ ∈ ܚ ، ܣ ≠ - ٢ } → ܚ

ܕ (ܣ) = ܣ + ٥

ܘܦܘܫܩܐ ܕܐܗܐ ܐܝܬܐ ܗܘܐ ܒܐܣܟܡܐ ܚܙܝ ܐܣܟܡܐ (٢ ـ ٣)



ܐܣܟܡܐ (٣ ـ ٢)

ܘܡܢ ܚܘܝܕܐ ܕܗܘܫܒܢܐ ܕܐܝܬܐ

ܣ	١,٩-	١,٩٩-	١,٩٩٩-	١,٩٩٩٩-	←	٢-
ܕ(ܣ)	٣,١	٣,٠١	٣,٠٠١	٣,٠٠٠١	←	٣

ܐܝܟ ܗܘܐ ܕܟܡܐ ܣ ← -٢ ܦܫ ܣ ܡܬܩܪܒܐ ܠܡܬܚܡܐ ܕܬܪܬܐ ܒܗܘܢ

ܣ) ← ٣

ܐܡܬܚܡܐ

$$\underset{ܣ \to -٢}{ܢܗ} \; ܕ(ܣ) = \underset{ܣ \to -٢}{ܢܗ} \; (ܣ + ٥) = ٣$$

ܐܝܬ ܝܠܗ ܡܬܚܡܐ ܠܡܢܝܢܐ ܕܡܬܩܪܒܐ (-٢) ܒܠܚܘܕ

ܦܬܐ (٤)

ܗܘܐ ܕ : ܚ ← ܚ ܐܝܬ ܐܝܟ

$$ܕ(ܣ) = \begin{cases} ١ & ܠܟܠ \; ܣ \geq ٠ \\ -١ & ܠܟܠ \; ܣ < ٠ \end{cases}$$

ܐܝܬ ܕܗܐ ܐܝܬ ܗܘܐ ܒܨܘܪܬܐ ܐܝܟ ܨܘܪܬܐ (٢ - ٣)

ܨ

ܨܘܪܬܐ (٢ - ٣)

ܐܝܟ ܐܝܬ ܐܝܟ ܒܡܬܚܡܐ ܠܡܬܩܪܒܐ ܡܢܝܢܐ ܝܠܗ ܡܢܗ ܕ(ܣ) ܐܝܟ

ܡܢܝܢܐ ܢܣܒ ܘܢܣܒ ܡܢ ܣܛܪܐ ܘܡܬܩܪܒܐ ܕܡܩܪܒ ܕܐܝܬ :

......	......	→	٠	←	......	......	...		
١	١	→		١ ←	١	١	١		

ܗܘܐ ܕܐܝܟ ܣ ܡܢܝܢܐ ܡܢ ܣܛܪܐ ܡܢ ܝܡܝܢܐ ܕܣ ܡܢܝܢܐ ܡܢ (١)

$\underset{ܣ \to ٠^{+}}{ܢܗ} \; ܕ(ܣ) = ١$

ܡܢܝܢܐ ܡܢ ܣܛܪܐ ܡܢ ܣܡܠܐ ܕ(ܣ) ܡܢܝܢܐ ܡܢ (-١)

$\underset{ܣ \to ٠^{-}}{ܢܗ} \; ܕ(ܣ) = -١$

ܐܝܟ ܕܗܐ ܐܡܬܚܡܐ ܝܠܗ ܠܐ ܡܬܚܡܐ ܡܬܩܪܒܐ ܡܢܝܢܐ ܡܢ ܣܛܪܐ ܘܡܢ ܡܬܩܪܒ ܕܐܝܬ ܠܡܬܚܡܐ ܚܕܐ ܐܝܟ ܣ ܡܢܝܢܐ ܡܢ ܣܛܪܐ.

ܐܘ ܠܡܬܚܡܐ ܕܚܕܐ ܝܠܗ ܡܢܝܢܐ ܚܕ ܕܐܝܬ ܐܝܟ ܡܬܩܪܒ ܡܬܚܡܐ ܡܬܚܡܐ ܕܚܕܐ ܕܐܝܬ ܡܬܚܡܐ ܡܢ ܡܬܚܡܐ.

ܘܐܦ ܓܘܦܐ ܕܟܠ ܕܟܠ ܚܠܝܦ ܒܟܘܠܝ ܟܠܝܘܕ ܐܢܐ ܕܡܫܘܚܬܐ ܠܐܢܐ ܕܠܐ ܕܡܢܐ ܕܣܟܠܗ ܐܘ ܣܟܠܗ ܒܡܫܘܚܬܐ ܘܡܢܝܢ ܕܣܟܠܐ ܝܠܗ ܣܟܠܐ ܒܝܢ ܐܘܚܕ ܠܐܢܐ ܝܘܡܢܐ ܘܣܟܠܐ ܒܡܫܘܚܬܐ ܝܠܗ (ܒ).

ܦܢܝܬܐ (٢ - ٣) ܡܫܘܢܝܢܘܬܐ ܕܣܟܐ

ܒܦܢܐ ܕܐܢܐ ܒܝܢ ܕܡܢܝܢ ܚܙܝܐ ܕܝܢܕܥܬ ܡܫܘܢܝܢܘܬܐ ܕܣܟܐ ܕܠܐ ܓܠܝܬܐ ܠܡܫܘܬܐ ܕܐܝܬ ܡܫܘܢܝܢܘܬܐ.

ܡܫܘܢܝܬܐ (١) ܚܕܝܢܘܬܐ ܕܣܟܐ :

ܐ. ܐܢ ܗܘܐ ܣܟܐ ܕ(ܣ) = ܠ١ (ܐܝܟ) ، ܣܟܐ ܕ(ܣ) = ܠ٢ (ܐܝܟ)
ܣ ← ܐ ܣ ← ܐ

ܗܝܕܝܢ ܠ١ = ܠ٢

ܗܢܐ ܡܥܢܐ : ܣܟܐ ܕܐܝܬ ܚܕ ܡܢܝܢܐ ܡܫܡܠܝܐ ܐ. ܐܝܟ . ܒܡܫܘܚܬܐ ܝܠܗ ܠܡܫܘܢܝܬܐ

ܡܫܘܢܝܬܐ (٢) :

ܣܟܐ ܕܐܢܐ ܡܫܘܚܬܐ ܚܕ ܡܢܝܢܐ ܡܫܡܠܝܐ ܐܘ ܡܢܝܢܐ ܡܫܘܚܬܐ ܒܢܦܫܗ

ܗܢܐ ܡܥܢܐ : ܐܢ ܗܘܐ ܕ(ܣ) = ܒ ܒܕ ܠ ∋ ܒ ܒܗܘܐ

ܣܟܐ ܕ(ܣ) = ܣܟܐ ܒ = ܒ
ܣ ← ܐ ܣ ← ܐ

ܓܘܦܐ

(١) ܣܟܐ (٥) = ٥
ܣ ← ٣

(٢) ܣܟܐ (٥) = ٥
ܣ ← $\frac{-1}{2}$

(٣) ܣܟܐ (٣ ܓ - ٤) = ٣ ܓ - ٤ ܒܕ ܐ ∋ ܒ
ܣ ← $\sqrt{2}$

(٤) ܣܟܐ (٥ ܣ + ٧) = ٥ ܣ + ٧ ܒܕ ܣ ∋ ܒ
ܣ ← ∆ ܣ .

ܡܫܘܢܝܬܐ :

ܢܦܩܬ ܒܡܢܝܢܐ ܕܐ، ܐܢܘܬ ܕ : ܒ ← ܒ ܐܢܐ ܐܝܬ

ܕ (ܣ) = ܐ . ܣ^{ܢ} + ܐ$_{ܢ-١}$ ܣ$^{ܢ-١}$ + ... = ܐ. ܒܕ ܐ. ، ... ، ܐ$_{ܢ-١}$ ، ܐ . ܡܢܝܢܐ

ܒܢܝܢܐ ـ ٠ ܦ ܒܡܫܡܠܝܐ ܡܢܝܢܐ ܕ ܐܢܐ ܡܫܘܚܬܐ ܡܢ ܕܪܓܐ ، ܘܐܢ ܗܘܐ ܣ : ܦ

ܟ ܐܢ̈ܐ ܐܝܟܐ ܦ ⊃ ܟ ܘܠܟܠ ܣ ∈ ܦ ، ܗ(ܣ) = ܕ(ܣ) ܦܬܐ ܐܡܝܢܬܐ
ܕܝܡܬܐ ܚܒܝܫܝ ܗܟܐ

ܣܘܟܡܬܐ (٣)

(١) ܐܢ̈ܐ ܚܒܝܫܝ ܗܟܐ ܐܝܬܘܗܝ ܥܡܐ ܐܝܟ ܟܠ ܡܢܝܢܐ ܚܕܝܕܐ ܡܫܚܠܦܐ ܘܡܫܘܚܬܐ ܕܐܘܪܐ ܡܢܝܢܐ ܒܚܒܘܫܝܐ ܕܐܡܝܢܘܬܐ ܕܐܡܝܢܘܬܐ.

ܗܢܐ ܘܕܝܢ : ܐ. ܐܘܗܝ ܕ(ܣ) ܐܝܟ ܕܐܡܝܢܘܬܐ ܠܐܢ̈ܐ ܚܒܝܫܝ ܗܟܐ ܒܗܘܐ

$$\lim_{ܣ \to ܐ} ܕ(ܣ) = ܕ(ܐ)$$

(٢) ܐ، ܐܘܗܝ ܗ(ܣ) ܐܝܟ ܕܐܡܝܢܘܬܐ ܠܐܢ̈ܐ ܠܕܝܡܬܐ ܕܐܢ̈ܐ ܚܒܝܫܝ ܗܟܐ ܐܝܟ ܕ(ܣ) ، ܗ ܡܫܬܕܠܬܐ ܠܗ ܚܕ ܡܢ ܚܒܘܫܬܐ *
ܕܡܢܝܢܐ ܚܕܝܕܐ (ܐ) ܒܗܘܐ

$$\lim_{ܣ \to ܐ} ܗ(ܣ) = \lim_{ܣ \to ܐ} ܕ(ܣ) = ܕ(ܐ)$$

ܘܦܬܐ (٦)

$$\lim_{ܣ \to \sqrt{٢}} (٣ܣ^{٤} - ٥ܣ^{٢} + ٢)$$

$$= ٣(\sqrt{٢})^{٤} - ٥(\sqrt{٢})^{٢} + ٢$$

$$= ١٢ - ١٠ + ٢ = ٤$$

ܘܦܬܐ (٧)

ܚܒܘܫ ܗܘܐ ܠ : ܚ/\{٣\} → ܚ ܐܢ̈ܐ ܐܝܟܐ $ܠ(ܣ) = \frac{ܣ^{٣} - ٢٧}{ܣ - ٣}$

ܐܚܘܝ $\lim_{ܣ \to ٣} ܠ(ܣ)$

ܣܘܟ ܗܘܝ ܕܐܢ̈ܐ ܠܐ ܝܠܗ ܡܫܬܕܠܬܐ ܐܝܟ ܣ = ٣ ܐܠܐ ܡܨܝ
ܕܚܘܝ ܐܢ̈ܐ ܐܝܟ ܕ : ܚ → ܚ ܐܝܟܐ ܕ(ܣ) =

$$= \frac{ܣ^{٣} - ٢٧}{ܣ - ٣} = \frac{(ܣ - ٣)(ܣ^{٢} + ٣ܣ + ٩)}{ܣ - ٣}$$

ܗܢܐ ܘܕܝܢ $ܕ(ܣ) = ܣ^{٢} + ٣ܣ + ٩$

ܠ ܕܘܟܐ ܦܫܝܛܐ ܚܕ ܡܢܝܢܐ (ܐ) ܦܝܫ ܡܕܝܐ ܚܒܘܫܬܐ ܕܡܢܝܢܐ ܐ

ܘܠܗܘ ܪܐܝ ܣܝܢܝܐ ܣܗܠ. ܠ(ܣ) ܝܠܗܘ ܪܐܝ ܠܟܣܡܝܐ ܠܪܐܝܐ

ܕ(ܣ) : ܘܟܠ ܕܪܗܪ ܟܗܘܪ

$$\lim_{ܣ \to ٣} ܠ(ܣ) = \lim_{ܣ \to ٣} ܕ(ܣ) = ܕ(٣)$$

$$= ٣^{٢} + ٣ \times ٣ + ٩$$

$$= ٩ + ٩ + ٩ = ٢٧$$

ܡܬܠܐ (٨)

ܟܢܝܫ ܗܘܐ ܕ : ܚ ← ܚ ܪܐܝ ܪܡܙܐ

ܕ(ܣ) = { ٢ܣ + ١ ܠܟܠ ܣ > ١ ; ٣ܣ² − ٢ ܠܟܠ ܣ ≤ ١ }

ܐܫܟܚ ܐܢ ܗܘܐ ܡܢܝܢܬܐ ܟܠ ܡܢ $\lim_{ܣ \to ٤}$ ܕ(ܣ) .

$$\lim_{ܣ \to ١} ܕ(ܣ) \quad , \quad \lim_{ܣ \to -٣} ܕ(ܣ)$$

ܢܦܩܬܐ ܕܝܢܬܐ ܠܪܐܝ ܝܠܗ ܒܪܗܘܡܐ ܟܝܢܬܐ ܐܫܟܠܐ (٢ ـ ٤)

ܐܫܟܠܐ (٢ ـ ٤)

ܚܙܝܐ ܗܘܝܐ ܕܟܡܐ ܕ ܣ ܗܘܝܐ ܡܩܪܒܬܐ ܩܪܝܒܐ ܡܢ ܡܢܝܢܐ (٤) ܒܗܘܝܐ ܕ(ܣ)
ܡܬܩܪܒܬܐ ܡܢ ܓܘ ܦܘܡܐ ܕ(ܣ) = ٢ܣ + ١ ܘܥܠ ܕܐܗܐ :

$$ܬܒ_{ܣ \to ٤}\ ܕ(ܣ) = ܬܒ_{ܣ \to ٤}\ (٢ܣ + ١) = ٢ \times ٤ \times ١ = ٩$$

(٧) ܟܕ ܣ ܡܩܪܒܐ ܠܘܬ ܡܢ ܡܢܝܢܐ (١) ܚܙܝܐ ܗܘܝܐ :

ܐܢܟܢ ܣ → ١⁺ ، ܗܕܐ ܐܘܟܝ ܣ > ١ ܗܘܝܐ ܕ(ܣ) ܡܬܩܪܒܬܐ ܡܢ ܓܘ ܦܘܡܐ

ܕ(ܣ) = ٢ܣ − ١

ܗܕܐ ܐܘܟܝ : $ܬܒ_{ܣ \to ١^+}\ ܕ(ܣ) = ܬܒ_{ܣ \to ١^+}\ (٢ܣ + ١) = ٢ \times ١ + ١ = ٣$ (ܚܕܐ ܡܢ ܝܡܝܢܐ)

ܘܐܢܟܢ ܣ → ١⁻ ܗܕܐ ܐܘܟܝ ܣ < ١ ܗܘܝܐ ܕ(ܣ) ܡܬܩܪܒܬܐ ܡܢ ܓܘ ܦܘܡܐ

ܕ(ܣ) = ٣ܣ² − ٢

ܗܕܐ ܐܘܟܝ : $ܬܒ_{ܣ \to ١^-}\ ܕ(ܣ) = ܬܒ_{ܣ \to ١^-}\ (٣ܣ^٢ − ٢) = ٣ \times ١^٢ − ٢ = ١$ (ܚܕܐ ܡܢ ܣܡܠܐ)

ܚܙܝܐ ܗܘܝܐ ܕܚܕܐ ܡܢ ܝܡܝܢܐ ≠ ܚܕܐ ܡܢ ܣܡܠܐ ܘܥܠ ܕܐܗܐ ܒܗܘܐ $ܬܒ_{ܣ \to ١}$
ܕ(ܣ) ܠܐ ܡܬܩܪܒܬܐ (ܐܘ ܠܝܬ ܐܘ ܦܝܫܐ ܡܣܬܪܩܬܐ ܠܝܬ ܠܗܿ ܐܬܐ ܚܕܐ ܓܒܐ (١))
ܒܗܝ ܚܕܐ ܓܒܐ ܡܢܝܢܐ ܬܪܝܨܐ ܡܫܚܠܦܐ . ܐܝܬ ܠܗ ܠܘܬܗ.

(٨) ܠܡܬܟܫܦܬܐ $ܬܒ_{ܣ \to −٣}$ ܕ(ܣ) ܚܙܝܐ ܗܘܝܐ ܐܢܟܢ ܣ → −٣ (ܡܢ ܝܡܝܢܐ ܐܘ ܡܢ
ܣܡܠܐ) ܒܗܘܝܐ ܕ(ܣ) ܡܬܩܪܒܬܐ ܡܢ ܓܘ ܦܘܡܐ ܕ(ܣ) = ٣ܣ² − ٢

ܘܥܠ ܕܐܗܐ : $ܬܒ_{ܣ \to −٣}\ ܕ(ܣ) = ܬܒ_{ܣ \to −٣}\ (٣ܣ^٢ − ٢) = ٣ \times (−٣)^٢ − ٢ = ٢٧ − ٢ = ٢٥$

ܡܦܩܬܐ (٩)

ܒܟܦܫ ܗܘܝܐ ܕ : ܪ → ܪ ܐܝܟ ܐܝܟܐ

$$ܕ(ܣ) = \begin{cases} ܣ^٢ & ܠܟܠ\ ܣ < ٢ \\ ٦ − ܣ٢ & ܠܟܠ\ ܣ \geq ٢ \end{cases}$$

ܦܫܘܩ ܗܘܝܬܐ $ܬܒ_{ܣ \to ٢}$ ܕ(ܣ)

...ܩ ܚܙܝ ܠܦܬܓܡܐ ܕܡܬܒܬܐ ܠܐܬܐ ܐܝܟ ܡܬܚܙܝܢܐ ܒܨܘܪܬܐ (٢ ـ ٥)

...ܐ ܗܘܝܐ ܕܐܢܟܢ ܣ → ٢ ܘܐܢܟܢ :

... → ٢⁺ ܗܕܐ ܐܘܟܝ ܣ > ٢ ܒܗܘܝܐ ܕ(ܣ) ܡܬܩܪܒܬܐ ܡܢ ܓܘ ܦܘܡܐ ܕ(ܣ) = ٦ − ܣ

...ܠ ܕܐܗܐ :

ܚܕܐ ܡܢ ܝܡܝܢܐ (ܗܕܐ ܐܘܟܝ ܐܢܟܢ ܣ → ٢⁺) ܒܗܘܐ

$$ܬܒ_{ܣ \to ٢^+}\ ܕ(ܣ) = ܬܒ_{ܣ \to ٢^+}\ (٣ܣ^٢ − ٢) = (٦ − ܣ) = ٦ − ٢ = ٤ = ܠ،$$

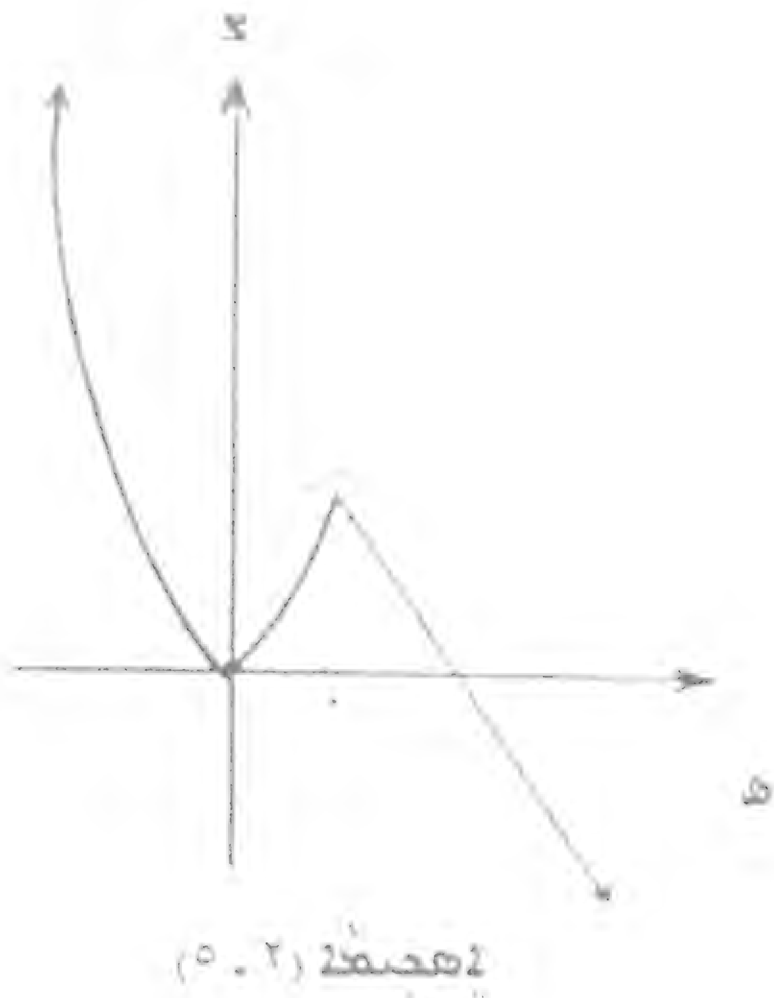
ܘܐܢܟܢ ܣ → ٢⁻ ܗܕܐ ܐܘܟܝ ܣ < ٢ ܗܘܝܐ ܕ(ܣ) ܡܬܩܪܒܬܐ ܡܢ ܓܘ ܦܘܡܐ

ܕ(ܣ) = ܣ² ܘܥܠ ܕܐܗܐ : ܚܕܐ ܡܢ ܣܡܠܐ (ܗܕܐ ܐܘܟܝ ܐܢܟܢ ܣ → ٢⁻)

ܗܘܐ $\lim_{ܗ \to ٢^-} ܕ(ܗ) = \lim_{ܗ \to ٢^-} ܗ^٢ = ٢^٢ = ٤ = ܠ$

ܘܟܕ ܚܬܡܐ ܡܢ ܝܡܝܢܐ = ܚܬܡܐ ܡܢ ܣܡܠܐ

ܒܗܘܐ : $\lim_{ܗ \to ٢} ܕ(ܗ) = ٤$ ܐܝܬ



ܐܣܟܡܐ (٢ ـ ٥)

ܡܬܠܐ (١٠)

ܐ، ܐܢ ܗܘܐ $ܕ(ܗ) = ܗ^٢ - ٣ܗ + ٥$ ܐܫܟܚ

(ܐ) $$\frac{ܕ(ܗ + \Delta ܗ) - ܕ(ܗ)}{\Delta ܗ}$$

(ܒ) $$\lim_{\Delta ܗ \to ٠} \frac{ܕ(ܗ + \Delta ܗ) - ܕ(ܗ)}{\Delta ܗ}$$

ܫܪܝܐ :

(ܐ) $ܕ(ܗ + \Delta ܗ) = (ܗ + \Delta ܗ)^٢ - ٣(ܗ + \Delta ܗ) + ٥$

$= ܗ^٢ + ٢ܗ\Delta ܗ + (\Delta ܗ)^٢ - ٣ܗ - ٣\Delta ܗ + ٥$

$ܕ(ܗ + \Delta ܗ) - ܕ(ܗ) = ܗ^٢ + ٢ܗ\Delta ܗ + (\Delta ܗ)^٢ - ٣ܗ - ٣\Delta ܗ$

$+ ٥ - (ܗ^٢ - ٣ܗ + ٥)$

$= ܗ^٢ + ٢ܗ\Delta ܗ + (\Delta ܗ)^٢ - ٣ܗ - ٣\Delta ܗ + ٥ - ܗ^٢ + ٣ܗ - ٥$

$= ٢ܗ\Delta ܗ + (\Delta ܗ)^٢ - ٣\Delta ܗ$

$$= \Delta ܣ \left[٢ ܣ + \Delta ܣ - ٣ \right]$$

$$\frac{ܕ(ܣ + \Delta ܣ) - ܕ(ܣ)}{\Delta ܣ} = \frac{\Delta ܣ \left[٢ ܣ + \Delta ܣ - ٣ \right]}{\Delta ܣ} = ٢ ܣ + \Delta - ٣$$

(ܒ) $$\lim_{\Delta ܣ \to ٠} \frac{ܕ(ܣ + \Delta ܣ) - ܕ(ܣ)}{\Delta ܣ}$$

$$= \lim_{\Delta ܣ \to ٠} (٢ ܣ + \Delta ܣ - ٣)$$

$$= ٢ ܣ + ܣܦܝܪܐ - ٣ = ٢ ܣ - ٣$$

ܚܘܫܒܬܐ (٤)

ܐܢ ܗܘܐ $\lim_{ܣ \to ܐ} ܕ(ܣ) = ܠ_1$ ܐܝܟ.

$\lim_{ܣ \to ܐ} ܗ(ܣ) = ܠ_2$ ܢܗܘܐ

(١) $$\lim_{ܣ \to ܐ} (ܕ \mp ܗ)(ܣ) =$$

$$\lim_{ܣ \to ܐ} ܕ(ܣ) \mp \lim_{ܣ \to ܐ} ܗ(ܣ) = ܠ_1 \mp ܠ_2$$

ܢܐ ܘܕܝܢ:

ܢܗܝܐ ܕܟܘܢܫܐ (ܐܘ ܦܘܪܫܐ) ܕܬܪܝܢ ܐܘ ܝܬܝܪ ܡܢ ܐܬܠܝܬ ܢܗܝܐ ܐܝܟ ܡܢܝܢܐ ܡܬܚܫܒ

ܘܐ ܟܘܢܫܐ (ܐܘ ܦܘܪܫܐ) ܕܬܪܘܢܗܘܢ ܢܗܝܐ ܐܝܟ ܕܐܝܬ ܗܘܐ ܡܢܝܢܐ.

(٢) $$\lim_{ܣ \to ܐ} (ܕ \cdot ܗ)(ܣ) =$$

$$\lim_{ܣ \to ܐ} ܕ(ܣ) \times \lim_{ܣ \to ܐ} ܗ(ܣ) = ܠ_1 \times ܠ_2$$

ܢܐ ܘܕܝܢ:

ܢܗܝܐ ܕܦܠܚܬܐ ܕܟܘܦܦܐ ܕܬܪܝܢ ܐܘ ܐܬܠܝܬ ܢܗܝܐ ܐܝܟ ܡܢܝܢܐ ܡܬܚܫܒ ܗܘܐ

ܦܠܚܬܐ ܕܟܘܦܦܐ ܕܬܪܘܢܗܘܢ ܐܘܐ ܐܝܟ ܕܐܝܬ ܗܘܐ ܡܢܝܢܐ.

(٣) ܠܟܠ ܟ $\in$ ܪ (ܟ ܡܢܝܢܐ ܫܪܝܪܐ)

$$\lim_{ܣ \to ܐ} ܟ \times ܕ(ܣ) = ܟ \times \lim_{ܣ \to ܐ} ܕ(ܣ) = ܟ ܠ_1$$

ܢܐ ܘܕܝܢ: ܢܗܝܐ ܕܦܠܚܬܐ ܕܟܘܦܦܐ ܕܟܘܠܐ ܫܪܝܪܐ ܠܗ ܐܘ ܡܢ ܢܗܝܐ ܐܝܟ

ܡܢܝܢܐ ܡܬܚܫܒ ܗܘܐ ܦܠܚܬܐ ܕܟܘܦܦܐ ܕܟܘܠܐ ܫܪܝܪܐ ܠܗ ܢܗܝܐ ܕܐܬܐ ܐܝܟ

ܡܢܝܢܐ ܒܠܚܘܕ.

$$\lim_{ܣ\to ٢} (ܕ \div ܡ)(ܣ) = \frac{\lim_{ܣ\to ٢} ܕ(ܣ)}{\lim_{ܣ\to ٢} ܡ(ܣ)} = \frac{ܠ_١}{ܠ_٢}$$

ܟܕ ܐܝܬ ܡ(ܣ) $\neq$ ܣܦܪܐ . $\lim_{ܣ\to ٢}$ ܡ(ܣ) = $ܠ_٢ \neq$ ܣܦܪܐ

ܗܘܝ ܙܗܝܪ :

ܣܟ̈ܐ ܕܦܠܓ̈ܘܬܐ ܕܦܠܓܘܬܐ ܕܬܪ̈ܝܢ ܐܝܬܝܗ̇ ܗܝ ܣܟ̈ܐ ܠܚܕ ܡܢܝܢܐ ܡܫܚܢܐ ܗܘܐ ܦܠܓܘܬܐ ܕܦܠܓܘܬܐ ܕܬܪ̈ܘܝܗܝ ܣܟ̈ܐ ܕܐܝܬ ܠܚܕ ܡܢܝܢܐ ܟܕ ܟܕ ܐܝܬ ܕܣܟܐ ܕܐܝܬ ܡܦܠܓܢܘܬܐ ܠܐ ܗܘܐ ܣܦܪܐ

ܡܬܕܪܫܢܘܬܐ : ܚܘܝܐ ܐܝܟ ܦܫܝܛܐ ܠܡܫܟܚܘ ܠܡܫܘܚܬܐ (؛) ܠܡܢܝܢܐ ܡܫܚܢܐ ܡܢ ܬܪܝܢ ܕܐܝܬܠܗܘܢ ܣܟܐ ܗܘܝ ܙܗܝܪ ܠܟܠܚܕ ܡܢ ܗܠܝܢ ܬܪܝܢ

ܦܘܓܡܐ (١١)

ܐܢ ܗܘܝܐ ܕ(ܣ) = $(ܣ^٣ + ٢ܣ + ١)$ ،

ܠ(ܣ) = $(٢ܣ^٢ - ٣ܣ + ٤)$

ܐܫܟܚ $\lim_{ܣ\to -١} (ܕ \cdot ܠ)(ܣ)$

ܫܪܝܐ :

$$\lim_{ܣ\to -١} (ܕ \cdot ܠ)(ܣ) = \lim_{ܣ\to -١} ܕ(ܣ) \times \lim_{ܣ\to -١} ܠ(ܣ)$$

$$= \lim_{ܣ\to -١} (ܣ^٣ + ٢ܣ + ١) \times \lim_{ܣ\to -١} (٢ܣ^٢ - ٣ܣ + ٤)$$

$$= (-١ - ٢ + ١)(٢ + ٣ + ٤) = -٢ \times ٩ = -١٨$$

ܦܘܓܡܐ (١٢)

ܐܢ ܗܘܝܐ ܕ(ܣ) = $ܣ^٢ - ٢ܣ - ٣$. ܠ(ܣ) = $٣ܣ^٢ + ٧$

ܐܫܟܚ $\lim_{ܣ\to ٣} \left(\frac{ܕ}{ܠ}\right)(ܣ)$

ܫܪܝܐ :

$$\lim_{ܣ\to ٣} \left(\frac{ܕ}{ܠ}\right)(ܣ) = \frac{\lim_{ܣ\to ٣} ܕ(ܣ)}{\lim_{ܣ\to ٣} ܠ(ܣ)}$$

$$= \frac{\lim_{ܣ \to ٣} (ܣ^٢ - ٢ܣ - ٣)}{\lim_{ܣ \to ٣} (٣ܣ^٢ + ٧)}$$

$$= \frac{٩ - ٦ - ٣}{٢٧ + ٧} = \frac{ܣܦܪܐ}{٢٤} = ܣܦܪܐ$$

ܛܘܦܣܐ (١٣)

ܐܢ ܗܘܐ ܕ(ܣ) $= \frac{ܣ^٢ - ٢ܣ - ٣}{ܣ^٢ - ٩}$ ܐܫܟܚ $\lim_{ܣ \to ٣}$ ܕ(ܣ)

ܦܬܪܐ: ܐ. ܟܕ ܡܫܚܠܦܝܢܢ $\lim_{ܣ \to ٣}$ ܕ(ܣ) ܒܡܫܠܡܢܘܬܐ ܕܡܢܝܢܐ ܒܡܢܝܢܐ ܚܕ ܚܕ

$= \frac{٩ - ٦ - ٣}{٩ - ٩} = \frac{ܣܦܪܐ}{ܣܦܪܐ}$) ܘܗܘܐ ܠܐ ܝܠܗ ܡܩܝܡܢܐ ܘܟܠ ܕܗܘܐ ܗܟܝ ܡܢ

ܡܬܚܙܝܬܐ ܕܗܕܐ ܒܚܕ ܫܒܝܠܐ ܘܚܕܐ ܛܘܦܣܐ ܫܢܝܐ ܟܠ ܡܢ ܡܠܬܐ ܘܡܢܝܢܐ

ܕܗܘܐ ܒܫܢܝ ܠܟܠ

$$ܕ(ܣ) = \frac{(ܣ - ٣)(ܣ + ١)}{(ܣ - ٣)(ܣ + ٣)} = \frac{ܣ + ١}{ܣ + ٣}$$ ܘܟܠܗܘܢ ܐܦ ܒܗܘܢ

$$\lim_{ܣ \to ٣} ܕ(ܣ) = \lim_{ܣ \to ٣} \frac{ܣ + ١}{ܣ + ٣}$$

$$= \frac{٣ + ١}{٣ + ٣} = \frac{٤}{٦} = \frac{٢}{٣}$$

ܡܫܘܕܥܢܘܬܐ (٥):

ܐ. ܐܢ ܗܘܐ ܕ(ܣ) ܗܘܐ ܗܘ ܡܢܝܢܐ ܠܘܬ ܐ ، ܡܫܚܠܦܢܘܬܐ ...

ܒܗܘܐ $\lim_{ܣ \to ܐ} \sqrt{ܕ(ܣ)} = \sqrt{\lim_{ܣ \to ܐ} ܕ(ܣ)}$ ܒܬܪܝܬܐ ܕ

$\lim_{ܣ \to ܐ}$ ܕ(ܣ) $\geq$ ܣܦܪܐ ܐܝܟ ، ܡܫܚܠܦܢܘܬܐ ܗܘܐ ܘܐܚܪܢܐ

ܛܘܦܣܐ (١٤)

ܐܢ ܗܘܐ ܕ : $\{ ܣ : ܣ \in ܚ ، ܣ \geq -٢ \} \to ܚ$

ܘܗܘ ܕ(ܣ) $= \sqrt{٣ܣ + ٦}$ ܐܫܟܚ $\lim_{ܣ \to ١}$ ܕ(ܣ)

$$\lim_{ܣ \to ١} ܕ(ܣ) = \lim_{ܣ \to ١} \sqrt{٣ܣ + ٦} = \sqrt{\lim_{ܣ \to ١} (٣ܣ + ٦)}$$

$$= \sqrt{٣ \times ١ \times ٦} = \sqrt{٩} = ٣$$

ܡܬܠܐ (١٥)

ܐ. ܢܗܘܐ ܗ : $\{ ܣ : ܣ \in ܚ ، ܣ \geq -١ \} \to ܚ$

$ܗ(ܣ) = \sqrt{ܣ + ١}$ ܐܫܟܚ $\lim_{ܣ \to -١} ܗ(ܣ)$

$$\lim_{ܣ \to -١} ܗ(ܣ) = \lim_{ܣ \to -١} \sqrt{ܣ + ١} = \sqrt{\lim_{ܣ \to -١} (ܣ + ١)}$$

$$= \sqrt{-١ + ١}$$

$$= \sqrt{ܨܦܪܐ} = ܨܦܪܐ$$

ܡܬܠܐ (١٦)

ܬܗܘܐ ܗܘܝܐ ܕ : $\{ ܣ : ܣ > ٤ \} \to ܚ$ ܐܝܟ $ܕ(ܣ) = \dfrac{ܣ - ٤}{\sqrt{ܣ} - ٢}$

ܢܫܟܚ ܐܢ ܐܝܬܘܗܝ $\lim_{ܣ \to ١} ܕ(ܣ)$

ܥܢܝܐ :

ܐ. ܒܫܘܠܡ ܡܫܟܚܬܐ $\lim_{ܣ \to ٢} ܕ(ܣ)$ ܒܡܦܠܚܘܬܐ ܕܡܫܘܚܠܦܢܘܬܐ ܠܡܫܟܚܬܐ ܚܕ ܡܢܐ ܘܗܘܐ ܒܡܫܠܡܘܬܐ ܡܢ ܒܬܪ ܗ ܒܡܢܝܢܐ (٤) ܒܡܢܝܢ

$\dfrac{٤ - ٤}{\sqrt{٤} - ٢} = \dfrac{ܨܦܪܐ}{ܨܦܪܐ}$ ܘܗܘܐ ܠܐ ܝܠܗ ܡܫܝܕܐ ܘܟܠ ܕܗܘܐ ܘܠܐ

ܡܦܪܫܬܐ ܕܐܬܐ ܘܗܘܐ ܐܝܟ ܕܐܬܐ :

$$ܕ(ܣ) = \frac{ܣ - ٤}{\sqrt{ܣ} - ٢} = \frac{(\sqrt{ܣ} - ٢)(\sqrt{ܣ} + ٢)}{\sqrt{ܣ} - ٢}$$ (ܒܫܘܚܠܦܐ ܕܟܠܗ ܐܝܟ ܦܘܪܫܢܐ ܒܝܢ ܬܪܝܢ ܡܪܒܥܐ)

ܘܟܠ ܕܗܘܐ ܗܘܐ ܥܢܝܐ ܒܡܫܪܘܬܐ ܕܐܬܐ :

$$\lim_{ܣ \to ٤} ܕ(ܣ) = \lim_{ܣ \to ٤} \frac{ܣ - ٤}{\sqrt{ܣ} - ٢}$$

$$= \lim_{ܣ \to ٤} \frac{(\sqrt{ܣ} - ٢)(ܣ + ٢)}{\sqrt{ܣ} - ٢}$$

$$= \lim_{ܣ \to ٤} (\sqrt{ܣ} + ٢)$$

$= \sqrt{٤} + ٢ = ٢ + ٢ = ٤$ ܐܝܟ (ܡܫܝܕܐ)

ܕܘܪܫܐ (٢ - ١)

ܬ١ : ܓܘ ܟܠ ܚܕ ܡܢ ܕܐܬܝܐ ܐܫܟܚ ܣܟܐ ܕܠܒܬܐ

(١) $\lim_{ܣ \to -٢} (ܣ^{٤} - ٣ܣ^{٣} + ٥)$

(٢) $\lim_{ܣ \to ١^{+}} \frac{ܣ^{٢} + ٣}{ܣ^{٢} - ٥}$

(٣) $\lim_{ܣ \to -١} \sqrt{ܣ^{٢} - ٤ܣ + ٦}$

(٤) $\lim_{ܣ \to ٠} (\sqrt{٣ܣ^{٢} + ١} - ٥ܣ)$

(٥) $\lim_{ܣ \to -١} \frac{ܣ + ١}{ܣ^{٢} + ٣ܣ + ٢}$

(٦) $\lim_{ܣ \to ٢} \frac{ܣ^{٣} - ٨}{ܣ^{٢} - ٤}$

(٧) $\lim_{ܣ \to ١} \frac{ܣ - ١}{\sqrt{ܣ} - ١}$

ܬ٢ : ܬܚܘܡ ܗܘܐ ܕ : ܬ ← ܬ ܐܝܟܐ $ܕ(ܣ) = \begin{cases} ܣ^{٢} + ١ & ܠܟܠ\ ܣ > ١ \\ ١ - ܣ & ܠܟܠ\ ܣ \leq -١ \end{cases}$

ܐܫܟܚ ܐܢ ܐܝܬܘܬܐ $\lim_{ܣ \to -١} ܕ(ܣ)$

ܬ٣ : ܬܚܘܡ ܗܘܐ ܕ : ܬ ← ܬ ܐܝܟܐ $ܕ(ܣ) = \begin{cases} ٣ܣ^{٢} - ١ & ܠܟܠ\ ܣ < ١ \\ ٢ܣ + ٣ & ܠܟܠ\ ܣ > ١ \end{cases}$

ܐܢ ܐܝܬ ܠܗ ܣܟܐ ܟܕ ܣ ← ١ ܦܫܘܩ ܠܗ .

ܬ٤ : ܐܢ ܗܘܬ $ܕ(ܣ) = ܐܣ^{٢} + ٣$ ܘܗܘܐ

$\lim_{ܣ \to -١} ܕ(ܣ) = ٥$ ܡܢ ܝܠܗ ܛܝܡܘܬܐ ܐ.

ܬ٥ : ܐܢ ܗܘܬ $ܕ(ܣ) = ܐܣ^{٣} - ܒܣ + ٢$ ܘܗܘܐ

$\lim_{ܣ \to ٣} ܕ(ܣ) = ٦$.

(−١ ، ٩) ܡܬܦܪܫܐ ܠܥܘܩܒܐ ܕܐܬܐ = ، ܡܢ ܝܠܗ ܛܝܡܘܬܐ

ܐ . ܒ ∈ ܬ

ܦܫܘܛܐ ܕܠܒܝܬܐ

ܚܒܠܬܐ

ܡܫܘܚܬܐ - ܕܪܘܪܒܐ

ܦܣܩܐ (٣ - ١)

ܡܥܠܬܐ:

ܒܕܒܪ ܒܦܫܘܛܐ ܚܒܝܪܐ ܗܘܝܐ ܕܗܘܐ ܘܗܘܝܐ ܕܒܝܬܐ ܕܗܘܐ ܘܒܠܥܣ ܒܗܘ ܦܫܘܛܐ ܗܘܝܐ ܕܚܒܠܬܐ ܕܗܘܐ ܘܡܫܘܚܬܐ ܕܡܫܚܬܗ.

ܐܘ ܡܠܘܐܐ ܦܘܫܩ ܚܕ ܡܢ ܡܠܘܐܐ ܐܝܟܢܐ ܘܡܫܐ ܠܗ ܡܫܘܚܬܐ ܘܐܘܐ ܠܦܘܚܬܐ ܕܡܦܠܚܬܗ ܘܡܦܠܚܬܗ ܠܗ ܡܠܘܐܐ ܐܚܪܢܐ ܐܝܟ: ܦܠܓܘܬܐ ܕܡܫܬܢܝܢܐ ܠܗܘܐ ܒܕܒܪܬܐ ܘܡܫܘܚܬܐ ܕܡܫܚܬܐ ܕܗܘ ܡܫܬܢܝܢܐ ܘܡܫܚܬܐ ܕܗܘܠܬܗ ܘܐܦ ܚܒܝܪܬܐ ܕܡܠܘܠܘܬܐ ܕܠܘܩܒܐ (ܐܝܟ ܒܪܘܚܬܐ ܐܘܪܚܬܐ) ܒܙܒܢܐ ܡܢ ܙܒܢܐ ܕܝܘܡܢܐ. ܡܙܕܒܢ ܕܒܘܐܠܐ ܐܚܪܢܐ.

ܦܣܩܐ (٣ - ٢)

ܒܘܐܠܐ ܡܫܕܪܐ [ܡܫܚܬܐ ܕܠܢܝܬܐ ܕܡܫܬܢܝܢܐ ܠܡܩܒܠܐ ܕܗܘܐ ܠܝܬ ܢܘܩܙܬܐ ܠܠܗ]

ܕܒܪܫ ܡܢ ܕܒܪܗ. ܚܒܝܪܬܐ ܕܠܢܝܬܐ ܕܬܪܝܨܐ ܚܒܝܪܐ ܒܬܪܝܢ ܢܘܩܙܐ

(ܣ١ ، ܨ١) ، (ܣ٢ ، ܨ٢) ܦܝܫ ܡܘܕܥܝܢ ܒܐܝܢܘܬܐ

$$ܠܢܝܬܐ\ ܡ = \frac{ܨ_٢ - ܨ_١}{ܣ_٢ - ܣ_١}\quad ܒܬܢܐܝ\ ܣ_٢ \neq ܣ_١$$

ܘܡܠܬܐ ܕܬܪܝܨܐ ܚܒܝܪܐ ܠܢܝܬܗ ܡ ܘܡܥܒܪ ܒܢܘܩܙܬܐ (ܣ١ ، ܨ١) ܦܝܫ ܡܘܕܥܝܢ ܒܐܝܢܘܬܐ: $ܨ - ܨ_١ = ܡ\,(ܣ - ܣ_١)$

ܒܟܠ ܗܘܝܐ $ܨ = ܕ(ܣ)$ ܐܢܐ ܦܘܫܩܢܐ ܗܘܝܐ ܡܢ ܚܒܝܪܬܗ ܐܝܟ ܒܐܣܟܡܐ ܡܟܠ ܗܘܝܐ ܢܘܩܙܬܐ ܐ = (ܣ ، ܨ) ܡܫܚܬܐ ܠܡܩܒܠܐ ܕܗܘܐ ܘܠܗ ܡܫܬܢܝܢܐ ܠܡܩܒܠܐ ܕܗܘܐ ܠܝܬ ܢܘܩܙܬܐ ܐ ܘܠܐܘܐ ܡܫܬܢܝܢܐ ܠܢܝܬܐ ܡܫܚܬܐ (٢) (١)

ܨ

ܒ (ܣ٢ ، ܨ٢)

ܢܬܪܐ

ܐ (ܣ١ ، ܨ١)

ܡܨܠܝܢܐ

ܒ

ܢܬܪܐ

ܒ

ܠܣ

ܣ١ ܠܣ

ܣ

ܡܨܠܝܢܐ

ܠܣ

ܡܨܠܝܢܐ (ܢܬܪܐ)

ܠܣ

ܐܣܟܡܐ (٣ . ١)

ܘܟܝܘܡ ܗܘܐ ܢܘܩܙܐ ܒ = (ܣ٢ ، ܨ٢) ܢܘܩܙܐ ܐܚܪܬܐ ܥܠ ܥܩܡܐ ܕܐܠܐ ܒܗܘܐ

$$\text{ܨܠܝܐ ܕܢܬܪܐ ܐ}\overline{\text{ܒ}} = \frac{\text{ܨ}_2 - \text{ܨ}_1}{\text{ܣ}_2 - \text{ܣ}_1} = \frac{\text{ܕ}(\text{ܣ}_2) - \text{ܕ}(\text{ܣ}_1)}{\text{ܣ}_2 - \text{ܣ}_1}$$

ܟܝܘܡ ܗܘܐ $\text{ܣ}_2 - \text{ܣ}_1 = \Delta\text{ܣ} \Leftarrow \text{ܣ}_2 = \text{ܣ}_1 + \Delta\text{ܣ}$

$$\text{ܨܠܝܐ ܕܢܬܪܐ ܐ}\overline{\text{ܒ}} = \frac{\text{ܕ}(\text{ܣ}_1 + \Delta\text{ܣ}) - \text{ܕ}(\text{ܣ}_1)}{\Delta\text{ܣ}}$$

ܐ. ܟܕ ܡܩܪܒ ܠܢܘܩܙܐ ܐ ܡܫܬܢܝܐ ܥܠ ܥܩܡܐ ܕܐܠܐ ܘܡܬܩܪܒ ܕܢܘܩܙܐ ܒ ܡܬܩܪܒ ܢܬܪܐ ܥܠ ܥܩܡܐ ܕܐܠܐ ܡܬܬܢܐ ܡܨܠܝܢܐ ܡܢ ܢܘܩܙܐ ܐ ܗܘ ܕܡܬܩܪܒ ܕܗܘܐ ܢܘܩܙܐ ܐ ܒܗܘܝܐ

$\Delta\text{ܣ} \rightarrow$ ܩܛܝܢܐ ܘܒܐܗܐ ܐܫܬܘܝܬܐ ܡܬܩܢ ܢܬܪܐ ܐ ܒ̄ ܡܢ ܡܨܠܝܢܐ ܠܥܩܡܐ ܕܐܠܐ ܠܘܬ ܢܘܩܙܐ ܐ ܘܥܠ ܕܐܗܐ ܡܫܪܝܬܐ ܠܡܐܡܪܐ :

ܨܠܝܐ ܕܡܨܠܝܢܐ ܠܘܬ ܢܘܩܙܐ (ܣ١ ، ܨ١)

$$\text{ܕ}'(\text{ܣ}_1) = \lim_{\Delta\text{ܣ} \to 0} \frac{\text{ܕ}(\text{ܣ}_1 + \Delta\text{ܣ}) - \text{ܕ}(\text{ܣ}_1)}{\Delta\text{ܣ}}$$

(١) ܐ. ܦܫܛܠܗ ܡܘܟܚܢܐ ܡܨܠܝܢܐ ܠܥܩܡܐ ܕܐܠܐ ܠܘܬ ܢܘܩܙܐ ܡܬܚܡܬܐ ܠܥܩܡܐ . ܡܨܠܝܢܐ ܗܘܝܐ ܠܗܘܢܐ.

(٢) ܗܢܐ ܘܕܡ ܡܨܠܝܢܐ ܠܐ ܗܘܐ ܗܕܐ ܨܕܝܐ

ܡܘܦܩܐ (١)

ܐܫܟܚ ܠܢܛܝܐ ܕܡܩܒܠܬܐ ܕܥܩܘܡܐ $ܨ = ܕ(ܗ) = ܗ^{٢} - ٢ܗ + ٣$

ܠܘܬ $ܗ = ٢$ ܘ ܐܫܟܚ ܡܥܕܠܬܐ ܕܐܗܐ ܡܩܒܠܬܐ

ܫܪܝܐ :

$$ܨ = ܕ(ܗ) = ܗ^{٢} - ٢ܗ + ٣$$

$$ܨ = ܕ(٢) = ٤ - ٤ + ٣ = ٣$$

∴ ܢܘܩܛܐ ܝܠܗ (٢ ، ٣)

ܠܢܛܝܐ ܕܡܩܒܠܬܐ ܠܘܬ ܢܘܩܛܐ $(ܗ_{١} ، ܨ_{١})$

$$= \underset{Δܗ \to ٠}{ܢܒ} \frac{ܕ(ܗ_{١} + Δܗ) - ܕ(ܗ_{١})}{Δܗ}$$

∴ ܠܢܛܝܐ ܕܡܩܒܠܬܐ ܠܘܬ ܢܘܩܛܐ (٢ ، ٣)

$$= \underset{Δܗ \to ٠}{ܢܒ} \frac{ܕ(٢ + Δܗ) - ܕ(٢)}{Δܗ}$$

$$= \underset{Δܗ \to ٠}{ܢܒ} \frac{(٢ + Δܗ)^{٢} - ٢(٢ + Δܗ) + ٣ - ٣}{Δܗ}$$

$$= \underset{Δܗ \to ٠}{ܢܒ} \frac{٤ + ٤Δܗ + (Δܗ)^{٢} - ٤ - ٢Δܗ + ٣ - ٣}{Δܗ}$$

$$= \underset{Δܗ \to ٠}{ܢܒ} \frac{٢Δܗ + (Δܗ)^{٢}}{Δܗ}$$

$$= \underset{Δܗ \to ٠}{ܢܒ} \frac{\cancel{Δܗ}(٢ + Δܗ)}{\cancel{Δܗ}} = ٢$$

ܡܥܕܠܬܐ ܕܡܬܩܢܐ : $ܨ - ܨ_{١} = ܠ(ܗ - ܗ_{١})$

∴ ܡܥܕܠܬܐ ܕܡܩܒܠܬܐ : $ܨ - ٣ = ٢(ܗ - ٢)$

$$ܨ - ٣ = ٢ܗ - ٤$$

$$٢ܗ - ܨ - ١ = ܨܦܪܐ$$

ܦܸܣܩܵܐ (٣ - ٣) ܡܫܲܚܠܦܵܢܘܼܬܵܐ ܕܦܠܘܠܘܬܐ ܫܪܝܪܬܐ ܕܠܘܬܦܐ ܡܫܚܠܦܢܐ

ܐܝܬܠܝ ܠܘܬܦܐ ܘܢܝܫ ܠܗ ܣܝܡܐ ܡܬܝܕܥܐ ܐܝܟ ܕܗܘܐ ܡܘܫܚܠܦܘܬܐ (ܨ) ܣܝܡܘܬܐ ܕܡܫܚܠܦܢܐ ܒܗܕ (ܣ) ܣܝܡܘܬܐ ܕܘܬܢܐ ܬܪܝܨܬܐ ܡܢ ܢܘܩܙܬܐ ܡܬܚܠܦܬܐ ܢܘܩܙܬܐ ܒܐܬܪܘܬܐ

$ܨ = ܕ(ܣ)$

$ܣ_١$ = ܘܬܢܐ ܡܘܬܪܨܬܐ ܡܢ ܢܘܩܙܬܐ ܕܥܘܒܕܐ ܥܠ ܡܫܝܚܬܐ ܠܢܘܩܙܬܐ ܒ

$ܣ_٢$ = ܘܬܢܐ ܡܘܬܪܨܬܐ ܡܢ ܢܘܩܙܬܐ ܕܥܘܒܕܐ ܥܠ ܡܫܝܚܬܐ ܠܢܘܩܙܬܐ ܐ



$ܨ_٢ = ܕ(ܣ_٢)$ = ܡܘܫܚܠܦܘܬܐ ܡܢ ܢܘܩܙܬܐ ܕܥܘܒܕܐ ܥܠ ܢܘܩܙܬܐ ܒ

$ܨ_١ = ܕ(ܣ_١)$ = ܡܘܫܚܠܦܘܬܐ ܡܢ ܢܘܩܙܬܐ ܕܥܘܒܕܐ ܥܠ ܢܘܩܙܬܐ ܐ

ܐܗܐ ܠܘܬܦܐ ܡܫܬܢܐ ܠܢܘܩܙܬܐ ܐ ܒܗܕ $ܣ_١$ ܣܝܡܘܬܐ ܘܬܢܐ ܡܢ ܥܘܒܕܐ ܕܘܟܬܗ ܘܡܬܠܬܗ ܥܒܝܪܐ ܐ ܠܢܘܩܙܬܐ ܒ ܘܡܫܬܢܐ ܬܠܬܗ ܒܗܕ $ܣ_٢$ ܣܝܡܘܬܐ ܘܬܢܐ ܡܢ ܥܘܒܕܐ ܕܘܟܬܗ

ܘܟܠ ܕܐܗܐ ܡܘܫܚܠܦܘܬܐ ܡܢ ܐ ܠܒ $= ܨ_٢ - ܨ_١ = ܕ(ܣ_٢) - ܕ(ܣ_١)$

ܘܬܢܐ ܡܘܬܪܨܬܐ ܠܡܫܬܢܬܐ ܕܡܘܫܚܠܦܘܬܐ ܡܢ ܐ ܠܒ $= ܣ_٢ - ܣ_١ = \Delta ܣ$

$ܣ_٢ = ܣ_١ + \Delta ܣ$

ܘܟܠ ܕܐܗܐ ܡܨܥܐ ܕܦܠܘܠܘܬܐ =

$$= \frac{ܕ(ܣ_٢) - ܕ(ܣ_١)}{\Delta ܣ} = \frac{ܕ(ܣ_١ + \Delta ܣ) - ܕ(ܣ_١)}{\Delta ܣ}$$

ܘܟܕ $\Delta ܣ \rightarrow$ ܨܦܪܐ، ܢܘܩܙܬܐ ܒ ܒܗܘܝܐ ܩܪܝܒܬܐ ܩܪܝܒܐ ܡܢ ܐ ܘܡܨܥܐ ܕܦܠܘܠܘܬܐ ܗܘܐ ܩܪܝܒܐ ܩܪܝܒ ܡܢ ܦܠܘܠܘܬܐ ܫܪܝܪܬܐ ܠܘܬ ܢܘܩܙܬܐ ܐ ܘܟܠ ܕܐܗܐ ܡܦܝܫܬܐ ܗܟܢܐ ܠܡܟܬܒ:

> ܦܠܘܠܘܬܐ ܫܪܝܪܬܐ (ܕܦܦܚܬܐ) ܠܘܬ $ܣ_١$ =
>
> $$ܦ(ܣ_١) = \lim_{\Delta ܣ \rightarrow 0} \frac{ܕ(ܣ_١ + \Delta ܣ) - ܕ(ܣ_١)}{\Delta ܣ}$$

ܦܘܫܩܐ (٢)

ܦܘܫܩܐ ܕܝܢ ܗܘ ܫܩܘܠ ܡܕܝܢܬܐ ܠܦܘܬ ܐܫܬܘܚܕܐ $ܗ = ٣ܘ^{٢} + ٢ܘ + ١$ ܟܕ ܗ ܡܫܬܘܚܕܐ ܒܡܫܚܬܐ، ܘܓܒܝܐ ܒܕܘܟܬܐ. ܐܝܟܢ ܡܬܚܫܒܢܘܬܐ ܕܦܘܫܩܐ ܐܝܟ ܕܘܟܬܐ ܡܬܚܫܒܢܐ ܡܢ ܒܗܘܠܐ ܕܘܚܠܗ ܐܝܟܢ ܡܬܚܫܒܢܗ ܒܟܠ ܕܘܟܐ .

$$ܗ = ܕ(ܘ) = ٣ܘ^{٢} + ٢ܘ + ١$$

$$ܐܢ\ ܘ = ٣ \qquad ܗ = ܕ(٣) = ٢٧ + ٦ + ١ = ٣٤$$

ܡܢ ܡܢܕܥܬܐ :

$$(١)\quad ܗ(ܘ_{١}) = \underset{\Delta ܗ \rightarrow ٠}{ܢܣܒ} \frac{ܕ(ܘ_{١} + \Delta ܘ) - ܕ(ܘ_{١})}{\Delta ܘ}$$

$$ܗ(٣) = \underset{\Delta ܗ \rightarrow ٠}{ܢܣܒ} \frac{ܕ(٣ + \Delta ܘ) - ܕ(٣)}{\Delta ܘ}$$

$$ܗ(٣) = \underset{\Delta ܗ \rightarrow ٠}{ܢܣܒ} \frac{٣(٣ + \Delta ܘ)^{٢} + ٢(٣ + \Delta ܘ) + ١ - ٣٤}{\Delta ܘ}$$

$$= \underset{\Delta ܗ \rightarrow ٠}{ܢܣܒ} \frac{٢٧ + ١٨\Delta ܘ + ٣(\Delta ܘ)^{٢} + ٦ + ٢\Delta ܘ + ١ - ٣٤}{\Delta ܘ}$$

$$= \underset{\Delta ܗ \rightarrow ٠}{ܢܣܒ} \frac{٢٠\Delta ܘ + ٣(\Delta ܘ)^{٢}}{\Delta ܘ}$$

$$= \underset{\Delta ܗ \rightarrow ٠}{ܢܣܒ} \frac{\cancel{\Delta ܘ}\,[٢٠ + ٣\Delta ܘ]}{\cancel{\Delta ܘ}} = ٢٠$$

ܗ ܗܕܐ ܡܬܚܫܒܢܘܬܐ ܕܦܘܫܩܐ ܐܝܟ ܕܘܟܬܐ ܡܬܚܫܒܢܐ ܡܢ ܒܗܘܠܐ ܕܘܚܠܗ

(٢) ܡܬܚܫܒܢܘܬܐ ܐܝܟ ܟܠ ܓܒܝܐ = $ܗ(ܘ)$

$$= \underset{\Delta ܗ \rightarrow ٠}{ܢܣܒ} \frac{ܕ(ܘ + \Delta ܘ) - ܕ(ܘ)}{\Delta ܘ}$$

$$= \underset{\Delta ܗ \rightarrow ٠}{ܢܣܒ} \frac{٣(ܘ + \Delta ܘ)^{٢} + ٢(ܘ + \Delta ܘ) + ١ - (٣ܘ^{٢} + ٢ܘ + ١)}{\Delta ܘ}$$

$$= \underset{\Delta ܣ \to \cdot}{\text{ܢܗܝ}} \frac{٣ܘ^{٢} + ٦ܘ\Delta ܘ + ٣(\Delta ܘ)^{٢} + ٢ܘ + ٢\Delta ܘ + ١ - ٣ܘ^{٢} - ٢ܘ - ١}{\Delta ܘ}$$

$$= \underset{\Delta ܣ \to \cdot}{\text{ܢܗܝ}} \frac{٦ܘ\Delta ܘ + ٣(\Delta ܘ)^{٢} + ٢\Delta ܘ}{\Delta ܘ}$$

$$= \underset{\Delta ܣ \to \cdot}{\text{ܢܗܝ}} \frac{\Delta ܘ(٦ܘ + ٣\Delta ܘ + ٢)}{\Delta ܘ} = ٦ܘ + ٢ \text{ ܡ/ܕܩ ܡܠܘܠܘܬܐ ܕܠܘܩܒܠ ܟܠ ܕܩܩܐ}$$

<u>ܡܚܒܪܢܘܬܐ</u> : ܐܘܪܚܐ ܡܚܒܪܬܐ ܠܡܫܟܚܬܐ ܕܝܠܝܬܐ ܕܡܦܪܫܢܘܬܐ ܕܥܒܝܕܐ ܕܐܬܐ
ܝܠܝܬ ܢܘܩܙܬܐ ܡܬܚܫܒܬܐ ܐܘ ܡܫܟܚܬܐ ܕܡܠܘܠܘܬܐ ܢܝܫܢܝܬܐ
ܠܘܩܒܠ ܘܝܟ ܝܘ ܚܝܠܐ ܝܠܝܬ ܝܘ ܟܠ ܕܩܩܐ ܦܬܐ ܡܢܝܬܐ ܐܘܪܚܐ ܕܡܣܕܪܬܐ.

ܦܬܓܡܐ (٣ ـ ٤) ܚܫܒܬܐ ܕܐܬܐ ܝܠܝܬ ܢܘܩܙܬܐ

> ܡܣܕܪܬܐ :
>
> ܐܢܗܘ ܗܘܐ ܕ ܐܬܐ ܡܣܕܪܬܐ ܝܘ ܕܘܟܐ [ܐ ، ܒ] ، $ܣ_{١} \in (ܐ$ ، $ܒ)$ ، ܗܕܝܢ
>
> ܐܬܐ ܕ ܡܫܟܚܬܐ ܝܠܗ ܠܚܫܒܬܐ ܝܠܝܬ $ܣ = ܣ_{١}$
>
> $$\Leftrightarrow \underset{\Delta ܣ \to \cdot}{\text{ܢܗܝ}} \frac{ܕ(ܣ_{١} + \Delta ܣ) - ܕ(ܣ_{١})}{\Delta ܣ}$$
>
> ܘܬܪܓܡܢܘܬܐ ܕܗܕܐ ܕܐܗܐ ܐܝܬܐ . ܢܘܩܙܬܐ ܕܝܬܐ ܝܠܗ
> ܚܫܒܬܐ ܕܐܬܐ ܕ ܝܠܝܬ $ܣ_{١}$ ܘܦܬܐ ܕܡܝܬܐ ܒܪܡܙܐ $ܕ(ܣ_{١})$

<u>ܡܚܒܪܢܘܬܐ</u> :

١) ܐܢ ، ܐܬܐ ܕ ܗܘܐ ܡܫܟܚܬܐ ܠܚܫܒܬܐ ܝܠܝܬ $ܣ = ܣ_{١}$ ܒܗܘܐ

$$\Leftrightarrow \underset{\Delta ܣ \to \cdot}{\text{ܢܗܝ}} \frac{ܕ(ܣ_{١} + \Delta ܣ) - ܕ(ܣ_{١})}{\Delta ܣ}$$

٢) ܐܢ ، ܐܬܐ $ܨ = ܕ(ܣ)$ ܗܘܐ ܡܫܟܚܬܐ ܠܚܫܒܬܐ ܝܠܝܬ ܟܠܚܕ ܡܢܘܬܗ
ܕܚܒܝܠܗ ܦܬܐ ܡܢܝܬܐ ܐܬܐ ܡܫܟܚܬܐ ܠܚܫܒܬܐ.

(٢) ܐ. ܐܢܐ ܨ = ܕ(ܣ) ܡܬܠܝܬܐ ܠܣܬܠܐ . ܣܬܠܬܐ ܚܕ ܥܠ ܢܘܩܙܐ ܐܝܢ ܣ
ܦܝܬ ܕܡܫܘܐ ܠܗ ܒܕܡܘܬܐ $\text{ܕ}'(\text{ܣ})$ ܐܘ $\frac{\text{ܕ}\,\text{ܨ}}{\text{ܕ}\,\text{ܣ}}$ [ܘܦܝܬ ܡܕܐ ܐܢܐ ܨ ܚܠ ܐܢܐ ܣ]
ܐܘ ܒܕܡܘܬܐ $\text{ܨ}'$.

ܡܘܦܬܐ (٣)

ܐܫܟܚ ܡܘܢܐ ܨ = $\text{ܕ}(\text{ܣ}) = ٤\text{ܣ}^{٢} - ٥\text{ܣ} + ٦$ ܐܫܟܚ $\text{ܕ}'(\text{ܣ})$

ܦܬܪܐ :

$$\text{ܕ}'(\text{ܣ}_1) = \lim_{\Delta\text{ܣ}\to 0} \frac{\text{ܕ}(\text{ܣ}_1 + \Delta\text{ܣ}) - \text{ܕ}(\text{ܣ}_1)}{\Delta\text{ܣ}}$$

$$\text{ܕ}'(١) = \lim_{\Delta\text{ܣ}\to 0} \frac{\text{ܕ}(١ + \Delta\text{ܣ}) - \text{ܕ}(١)}{\Delta\text{ܣ}}$$

$$= \lim_{\Delta\text{ܣ}\to 0} \frac{٤(١ + \Delta\text{ܣ})^{٢} - ٥(١ + \Delta\text{ܣ}) + ٦ - (٤ - ٥ + ٦)}{\Delta\text{ܣ}}$$

$$= \lim_{\Delta\text{ܣ}\to 0} \frac{٤ + ٨\Delta\text{ܣ} + ٤(\Delta\text{ܣ})^{٢} - ٥ - ٥\Delta\text{ܣ} + ٦ - ٥}{\Delta\text{ܣ}}$$

$$= \lim_{\Delta\text{ܣ}\to 0} \frac{\cancel{\Delta\text{ܣ}}(٣ + ٤\Delta\text{ܣ})}{\cancel{\Delta\text{ܣ}}} = ٣$$

[ܘܐܝܬ ܕܡܫܠܡܐ $\Delta\text{ܣ} \neq$ ܨܦܪܐ]

ܡܘܦܬܐ (٤)

ܐܢ ܗܘܬ ܕ : ܚ / {٠} ← ܚ ܐܝܟܐ $\text{ܕ}(\text{ܣ}) = \frac{٣}{\text{ܣ}}$ ܐܫܟܚ $\text{ܕ}'(\text{ܣ})$ ؟

$$\text{ܕ}'(\text{ܣ}) = \lim_{\Delta\text{ܣ}\to 0} \frac{\text{ܕ}(\text{ܣ}_1 + \Delta\text{ܣ}) - \text{ܕ}(\text{ܣ})}{\Delta\text{ܣ}}$$

$$= \underset{\Delta ܣ \to .}{ܬܒ} \frac{\frac{٣}{ܣ + \Delta ܣ} - \frac{٣}{ܣ}}{\Delta ܣ}$$

$$= \underset{\Delta ܣ \to .}{ܬܒ} \frac{\frac{٣ܣ - ٣(ܣ + \Delta ܣ)}{ܣ(ܣ + \Delta ܣ)}}{\Delta ܣ}$$

$$= \underset{\Delta ܣ \to .}{ܬܒ} \frac{\frac{٣ܣ - ٢ܣ - ٣\Delta ܣ}{ܣ + \Delta ܣ}}{\Delta ܣ}$$

$$= \underset{\Delta ܣ \to .}{ܬܒ} \frac{-٣\Delta ܣ}{ܣ^{٢} + ܣ\Delta ܣ} \times \frac{١}{\Delta ܣ}$$

$$= \underset{\Delta ܣ \to}{ܬܒ} \frac{-٣}{ܣ^{٢} + ܣ\Delta ܣ} = \frac{-٣}{ܣ^{٢} + ܣ \times ܨܦܪܐ}$$

$$= \frac{-٣}{ܣ^{٢}}.$$

ܡܥܝܪܢܘܬܐ :

(١) ܐܢ ܗܘܬ ܨ = ܕ(ܣ) ܐܝܬ ܡܦܠܛܬܐ ܥܠ ܚܠܛܬܐ ܒܗܘܢ $ܨ' = ܕ'(ܣ) = \frac{ܕܨ}{ܕܣ}$
ܐܝܬ ܚܕܐ ܦܬܐ ܡܕܝܬܐ ܐܝܬ ܕܚܠܛܬܐ ܡܕܡܝܐ ܕܐܝܬ ܘܠܗ ܚܠܛܬܐ
ܕܐܝܬ ܐܚܪܬܐ ܕܗܘܢ.

(٢) ܟܕ $ܨ = ܕ(ܣ) = \frac{ܕܨ}{ܕܣ}$ ܐܝܬ ܚܕܐ، ܡܫܢܝܬܐ ܠܗ ܡܥܒܕܢܘܬܐ ܕܚܠܛܬܗ
ܘܦܝܫ ܡܫܝܐ ܠܚܠܛܬܗ ܐܝܬ ܕܚܠܛܬܐ ܡܕܡܝܐ ܒܕܡܘܬ $ܨ = ܕ(ܣ) = \frac{ܕ^{٢}ܨ}{ܕܣ^{٢}}$
ܘܠܗ ܐܝܬ ܚܕܐ ܦܬܐ ܒܘܡܗܐ ܐܝܬ ܕܚܠܛܬܐ ܡܚܪܬܐ ܕܐܝܬ.

(٣) ܐܢ ܗܘܬ ܨ = ܕ(ܣ) ܦܘܫܩܐ ܡܦܫܩܐ ܒܗܘܢ
ܐܢܫܝܐ ܕܡܬܚܫܒܢ ܠܡܦܫܩ ܕܐܝܬ ܐܢܫ ܥܠ ܢܘܣܟܐ ܨ = ܕ(ܣ) ܦܘܫܩܐ

ܥܠ ܢܩܘܕܐ [ܐܘ ܦܬܐ ܕܡܫܘܚܬܐ ܓܢܝܬܐ ܕܢܩܘܕܐ].

(ܗ) ܐ. ܢܗܘܐ ܡ = ܕ(ܘ) ܐܝܟ ܡ ܡܫܘܚܬܐ ، ܘ ܙܒܢܐ ܡܢ ܫܪܝܬܐ ܠܡܪܕܘܬܗ ܒܗܘܐ :

$$(\text{ܐ})\ \text{ܡ} = \text{ܕ}(\text{ܘ}) = \frac{\text{ܕ ܡ}}{\text{ܕ ܘ}} = \text{ܡܠܘܠܘܬܐ ܡܨܥܝܬܐ ܓܢܒ ܟܠ ܙܒܢܐ} = \text{ܡ}(\text{ܘ})$$

$$(\text{ܒ})\ \text{ܡ} = \text{ܕ}(\text{ܘ}) = \frac{\text{ܕ}^{2}\text{ ܡ}}{\text{ܕ ܘ}} = \text{ܡܘܪܗܒܢܐ ܓܢܒ ܟܠ ܙܒܢܐ} = \text{ܣ}(\text{ܘ})$$

ܗܢܐ ܟܕܡ :

ܡܠܘܠܘܬܐ ܡܨܥܝܬܐ ܡ(ܘ) = ܦܬܐ ܕܡܫܘܚܬܐ ܡܢܝܬܐ ܕܡܫܘܚܬܐ (ܒ) ܠܙܒܢܐ (ܘ)

ܡܘܪܗܒܢܐ ܓܢܒ ܟܠ ܙܒܢܐ ܣ(ܘ) = ܦܬܐ ܕܡܫܘܚܬܐ ܡܢܝܬܐ ܕܡܫܘܚܬܐ ܡ ܠܙܒܢܐ ܘ.

(ܘ) ܐܘܪܚܐ ܡܢܝܬܐ ܠܡܫܟܚܬܐ ܕܡܫܘܚܬܐ ܦܬܐ ܡܢܝܬܐ ܐܘܪܚܐ ܕܡܫܪܝܬܐ.

ܕܘܪܫܐ (٣ - ١)

ܬ١ : ܐ. ܢܗܘܐ ܕ(ܣ) : ٣ܣ^{2} - ٥ ܐܫܟܚ ܠܡܨܠܬܐ ܕܡܫܝܚܬܐ ܕܢܩܘܕܐ ܕ ܓܢܒ ܣ = +١

ܬ٢ : ܐ. ܢܗܘܐ ܕ(ܣ) = ܣ^{2} + ٦ܣ + ٧ ܐܫܟܚ ܕ(ܣ) ܡܢ ܐܫܟܚ ܠܡܨܠܬܐ
ܕܡܫܝܚܬܐ ܕܢܩܘܕܐ ܕ ܓܢܒ ܣ = -١

ܬ٣ : ܐ. ܐܫܟܚܬܐ ܡ = ܘ^{2} + ܘ + ٥ ܡܬܚܫܒܐ ܘܡܢܐ ܕܡܬܚܫܒܐ ܥܠ ܡܫܚܐ ܠܡܪܕܝܐ ܐܝܟ ܡ ܡܫܘܚܬܐ ܒܡܝܬܪܐ، ܘ ܙܒܢܐ ܒܙܒܢܐ ܡܢ ܫܪܝܬܐ ܠܡܪܕܘܬܗ. ܐܫܟܚ ܡܠܘܠܘܬܐ ܡܨܥܝܬܐ ܕܡܬܚܫܒܐ ܒܟܠ ܙܒܢܐ ܡܢ ܐܫܟܚ ܒܗܘܐ ܘܡܠܘܠܘܬܗ ܓܢܒ ܙܒܢܐ ܫܢܝܬܐ ܡܢ ܒܪܝܫܐ ܕܪܗܛܗ

ܬ٤ : ܐܫܟܚ ܫܪܝܠܬܐ ܩܕܡܝܬܐ ܕܐܝܬ ܠܟܠ ܡܢ ܗܠܝܢ ܕܠܬܚܬ :

$$(\text{ܐ})\quad \text{ܨ} = \frac{2}{\text{ܣ} - 1} \qquad \text{ܟܕ ܣ} \neq 1$$

$$(\text{ܒ})\quad \text{ܗ}(\text{ܣ}) = \text{ܣ}^{2} + 3\text{ܣ} + 5$$

ܦܣܩܐ (٣ - ٥) ܫܪܫܢܝܐ ܐܝܟ ܐܝܟܐ ܠܡܫܚܠܦܢܘܬܐ ܕܣܝܒܠܬܐ

ܡܢ ܗܪܟܐ ܠܘܬܢ ܗܘܐ ܡܫܚܠܦܢܘܬܐ ܕܣܝܒܠܬܐ ܒܝܕ ܢܡܘܣܐ ܡܫܚܠܦܢܘܬܐ ܒܫܡܐ ܗܕܐ ܠܟܠܗܘܢ

ܘܡܦܠܣܬܐ ܕܡܫܪܪܬܐ ܠܣܝܒܠܬܐ ܘܡܫܘܚܬܐ ܕܒܬܪ ܡܫܚܬܐ ܗܠܝܢ ܬܪܬܐ ܕܫܪܫܢܐ ܡܫܘܚܬܐ (ܐܣܐ) ܠܡܫܚܠܦܢܘܬܐ ܕܣܝܒܠܬܐ ܕܗܕܐ ܡܫܚܠܦܢܘܬܐ. ܐܢ ܡܫܘܚܬܐ ܡܫܪܪܐ ܡܢ ܡܫܚܠܦܢܘܬܐ ܕܣܝܒܠܬܐ ܕܪܫܐ ܡܢ ܗܕܐ ܒܦܠܘܛܘܬܐ ܘܒܦܫܝܛܘܬܐ ܕܟܠ ܕܪܫܐ ܠܡܫܪܪܘܬܐ ܘܟܠ ܦܬܓܡ ܡܫܘܚܬܐ ܕܐܢܐ ܐܣܐ ܘܒܟܠܗ ܒܪܫܐ ܕܐܣܐ ܘܟܠܗܝܢ ܕܡܦܠܣܬܗ ܡܢ ܟܠ ܛܘܦܣܐ.

ܐܣܐ (ܡܫܘܚܬܐ) ܩܕܡܝܐ :

ܗܕܐ ܡܫܚܠܦܢܘܬܐ ܡܫܠܡܬܐ ܠܣܝܒܠܬܐ ܘܣܝܒܠܬܗ ܗܘܐ ܣܦܪܐ

ܗܢܐ ܘܕܝܢ ܐܢ ܝܘܬܒ $ܨ = ܕ(ܣ) = ܟ$ ܟܕ $ܟ \in ܚ$ ܡܢܝܢܐ

$ܨ' = ܕ'(ܣ) = $ ܣܦܪܐ

ܛܘܦܣܐ :

١) $ܕ(ܣ) = ٣$ $\Leftarrow$ $ܕ'(ܣ) = $ ܣܦܪܐ

٢) $ܨ = -٢\sqrt{٧}$ $\Leftarrow$ $ܨ' = $ ܣܦܪܐ

٣) $ܡ = ܕ(ܘ) = \frac{٣}{ܛ}$ $\Leftarrow$ $ܡ' = ܕ'(ܘ) = $ ܣܦܪܐ

٤) $ܕ(ܣ) = ܐ$ $\Leftarrow$ $ܕ'(ܣ) = $ ܣܦܪܐ ܟܕ $ܐ \in ܚ$

ܐܣܐ ܬܪܝܢܐ :

ܐܢ ܗܘܐ $ܨ = ܕ(ܣ) = ܣ^{ܢ}$

ܟܕ $ܣ \in ܚ \setminus \{٠\}$

ܗܘܝܐ $ܨ' = \frac{ܕܨ}{ܕܣ} = ܕ'(ܣ) = ܢ\, ܣ^{ܢ-١}$

ܛܘܦܣܐ :

١) $ܕ(ܣ) = ܣ^{٤}$ $\Leftarrow$ $ܕ'(ܣ) = ٤\, ܣ^{٣}$

(٢) ܨ = $ܣ^{-٣}$ ⟸ $\frac{ܕܨ}{ܕܣ}$ = − ٣ $ܣ^{-٤}$ ܟܕ ܣ ≠ ٠

(٣) ܕ (ܣ) = $\frac{١}{\sqrt{ܣ}}$ ⟸ ܕ (ܣ) = $\frac{١}{ܣ^{\frac{١}{٢}}}$ = $ܣ^{-\frac{١}{٢}}$

∴ ܕ́(ܣ) = $-\frac{١}{٢}$ $ܣ^{-\frac{٣}{٢}}$ ܟܕ ܣ > ܣܦܝܪܐ

(٤) ܕ (ܣ) = ܣ ⟸ ܕ́ (ܣ) = ١ !

(٥) ܡ = ܕ (ܢ) = $\sqrt[٣]{ܢ}$ ⟸ ܡ = ܕ (ܢ) = $ܢ^{\frac{١}{٣}}$

∴ ܡ́ = ܕ́ (ܢ) = $\frac{ܕܡ}{ܕܢ}$ = $\frac{٢}{٣}$ $ܢ^{-\frac{١}{٣}}$

ܐܦܐ ܡܠܝܬܐ :

ܣܥܝܪܬܐ ܕܟܘܦܠܐ ܦܫܝܛܬܐ ܠܘ ܗܕܐ ܡܫܠܡܬܐ ܠܣܥܝܪܬܐ ܟܘܦܠܐ
ܟܘܦܠܐ ܦܫܝܛܬܐ ܠܘ ܣܥܝܪܬܐ ܕܗܕܐ.

ܗܢܐ ܘܕܝܢ ܐܢ ܗܘܬ ܕ(ܣ) = ܟ × ܠ(ܣ)

ܟܕ ܟ ܟܘܦܠܐ ܦܫܝܛܬܐ . ܠ ܗܕܐ ܡܫܠܡܬܐ ܠܣܥܝܪܬܐ ܒܗܘܐ

ܕ́ (ܣ) = ܟ × ܠ́ (ܣ) .

ܡܘܦܬܐ :

(١) ܕ (ܣ) = ٣ $ܣ^{٢}$ ⟸ ܕ́ (ܣ) = ٣ × ٢ ܣ

ܕ́ (ܣ) = ٦ ܣ

(٢) ܨ = ٥ $ܣ^{٤}$ ܐܫܟܚ ܨ́ ܟܕ ܣ = ١

ܨ = ٥ $ܣ^{٤}$ ⟸ ܨ́ = ٥ × ٤ $ܣ^{٣}$

ܨ́ = ٢٠ $ܣ^{٣}$

ܘܟܕ ܣ = ١ ܒܗܘܐ ܨ́ = ٢٠ × $١^{٣}$ = ٢٠

(٣) ܐܢ ܗܘܬ ܕ(ܣ) = $\frac{٣}{ܣ}$ ܟܕ ܣ ≠ ܣܦܝܪܐ ܐܫܟܚ

ܕ́ (ܣ) ، ܕ (ܣ)

ܚܠܐ :

ܕ(ܣ) = $\frac{٣}{ܣ}$ = ٣ $ܣ^{-١}$

$ܕ́(ܗ) = ٣ \times -٢\, ܗ^{-٣} = -٦\, ܗ^{-٣}$

$ܕ̈(ܗ) = -٦ \times -٣\, ܗ^{-٤} = ١٨\, ܗ^{-٤}$

[ܕ̈ (ܗ) ܝܠܗ̇ ܚܝܠܬܐ ܕ́(ܗ)]

ܡܫܪ̈ܝܘܬܐ :

ܠܒܪܡܬܐ ܕܚܘܠܫ̈ܐ ܘܝܕ̈ܥܐ ܦܝܫ ܚܫܝܒܐ ܠܘܩܒܠ ܡܢ̈ܝܢܐ ܦܫܝ̈ܛܐ ܐܝܟ ܐ ܗ (ܗ)

ܘܡܘܬܪܐ ܡܢ ܐܝܟ ܡܢ̈ܝܢܐ (١) ܘܦܝܫ ܡܫܝܢܐ ܦܠܓܐ ܚܕ ܕܪ̈ܐ

(٤) $ܕ(ܗ) = ٨\sqrt{ܗ} \Leftarrow ܕ(ܗ) = ٨\, ܗ^{\frac{١}{٢}}$ ، ܗ > ܣܦܝܪܐ

$\therefore ܕ́(ܗ) = 4\, ܗ^{-\frac{١}{٢}}$

(٥) ܐܢ ܗܘܬ $ܕ(ܗ) = ٥\, ܗ$ ܐܫܟܚ $ܕ́(ܗ)$ ، $ܕ̈(ܗ)$

ܫܪܝܐ :

$ܕ(ܗ) = ٥\, ܗ$

$ܕ́(ܗ) = ٥$ ܠܡܐ ؟

$ܕ̈(ܗ) =$ ܣܦܝܪܐ ܠܡܐ ؟

(٦) ܐܢ ܗܘܬ $ܕ(ܗ) = \frac{١}{٢}\, ܗ^{٦}$ ܐܫܟܚ $ܕ́(١)$ ، $ܕ̈(-١)$

$ܕ(ܗ) = \frac{١}{٢}\, ܗ^{٦} \Leftarrow ܕ́(ܗ) = ٣\, ܗ^{٥}$

$ܕ̈(ܗ) = ١٥\, ܗ^{٤}$

$\therefore ܕ́(١) = ٣ \times ١^{٥} = ٣$

$ܕ̈(-١) = ١٥ \times (-١)^{٤} = ١٥$

ܐܝܟ ܕܝܠܢܐ :

ܚܝܠܬܐ ܕܟܘܢܫܐ ܕܡܢ̈ܝܢܐ ܡܬܚܫܒܐ ܡܢ ܐܝܟ ܟܢܘܫܝܐ ܠܚܝܠܬܐ ܟܘܠ ܟܘܢ̈ܫܐ ܕܚܝܠܬܐ ܕܐܝܟ ܐܝܟ .

ܦܬܐ ܘܕܝܢ :

ܐܢ ܗܘܬ $ܕ(ܗ) = ܠ(ܗ) + ܦ(ܗ)$ ܒܥܘܕܐ)

$ܕ́(ܗ) = ܠ́(ܗ) + ܦ́(ܗ)$

ܠܘܦܬܐ :

(١) $ܨ = ٣\, ܗ^{٢} + ٥$

ܨ̇ = ܕܨ / ܕܣ = ٦ ܣ

(٢) ܐܢ ܗܘܬ ܕ(ܣ) = ٣ܣ^٣ − ٥ ܣ^٢ + ١/٢ ܣ + ٣

ܐܫܟܚ ܕ̇(ܣ) ، ܕ̈(ܣ)

ܫܪܝܐ

ܕ̇(ܣ) = ٩ ܣ^٢ − ١٠ ܣ + ١/٢

ܕ̈(ܣ) = ١٨ ܣ − ١٠

(٣) ܐܢ ܗܘܬ ܐ ، ܒ ، ܓ ∈ ܡ ، ܕ(ܣ) = ܐܣ^٤ + ܒ ܣ^٢ + ܓ

ܐܫܟܚ ܕ̇(ܣ) ، ܕ̈(ܣ)

ܫܪܝܐ :

ܕ(ܣ) = ܐ ܣ^٤ + ܒ ܣ^٢ + ܓ

ܕ̇(ܣ) = ٤ ܐ ܣ^٣ + ٢ ܒ ܣ

ܕ̈(ܣ) = ١٢ ܐ ܣ^٢ + ٢ ܒ

ܐܦ ܚܫܒܬܐ :

ܚܝܠܬܐ ܕܦܠܚܘܬܐ ܕܡܘܦܩܐ ܕܬܪܝܢ ܗܘܐ ܡܫܠܡܢܐ ܠܚܝܠܬܐ ܩܕܡܝܬܐ
ܬܢܝܢܐ ܡܕܡܝܐ × ܚܝܠܬܐ ܕܬܢܝܢܐ ܩܕܡܝܐ + ܬܢܝܢܐ ܩܕܡܝܐ ×
ܚܝܠܬܐ ܕܬܢܝܢܐ ܡܕܡܝܐ

ܗܢܐ ܘܕܝܢ

ܐܢ ܗܘܬ ܕ(ܣ) = ܠ(ܣ) × ܗ(ܣ) ܒܗܘܢܐ :

ܕ̇(ܣ) = ܠ(ܣ) × ܗ̇(ܣ) + ܗ(ܣ) × ܠ̇(ܣ)

ܡܬܠܐ :

(١) ܕ(ܣ) = (ܣ^٣ − ܣ + ٣)(ܣ^٢ + ٢ ܣ − ١)

ܫܪܝܐ :

ܕ̇(ܣ) = (ܣ^٣ − ܣ + ٣)(٢ ܣ + ٢)
+ (ܣ^٢ + ٢ ܣ − ١)(٣ ܣ^٢ − ١)

(٢) ܬ(ܝ) = √ܝ (ܝ + ٦) ܐܫܟܚ ܠܟܠ ܚܕ ܬ̇(ܝ) ، ܬ̈(ܝ)

ܫܪܝܐ :

ܬ(ܝ) = ܝ^{١/٢} (ܝ + ٦) = ܝ^{٣/٢} + ٦ ܝ^{١/٢}

$$\text{ܛ}(\text{ܢ}) = \frac{٣}{٢}\,\text{ܢ}^{\frac{١}{٢}} + ٣\,\text{ܢ}^{-\frac{١}{٢}} = \frac{٣}{٢}\sqrt{\text{ܢ}} + \frac{٣}{\sqrt{\text{ܢ}}}$$

$$\text{ܛ}'(\text{ܢ}) = \frac{٣}{٤}\,\text{ܢ}^{-\frac{١}{٢}} - \frac{٣}{٢}\,\text{ܢ}^{-\frac{٣}{٢}} = \frac{٣}{٤\,\text{ܢ}^{\frac{١}{٢}}} - \frac{٣}{٢\,\text{ܢ}^{\frac{٣}{٢}}}$$

$$= \frac{٣}{٤\sqrt{\text{ܢ}}} - \frac{٣}{٢\sqrt{\text{ܢ}^{٣}}}$$

(٣) $\text{ܛ}(\text{ܢ}) = \left(\text{ܢ} - \frac{١}{\text{ܢ}}\right)(\text{ܢ}^{٢} + ٣\text{ܢ} + ١)$ ܐܫܟܚ $\text{ܛ}'(\text{ܢ})$

ܦܢܝܐ :

$$\text{ܛ}(\text{ܢ}) = (\text{ܢ} - \text{ܢ}^{-١})(\text{ܢ}^{٢} + ٣\text{ܢ} + ١)$$

$$\frac{\text{ܕܛ}}{\text{ܕܢ}} = \text{ܛ}'(\text{ܢ}) = (\text{ܢ} - \text{ܢ}^{-١})(٢\text{ܢ} + ٣) + (\text{ܢ}^{٢} + ٣\text{ܢ} + ١)(١ + \text{ܢ}^{-٢})$$

$$= \left(\text{ܢ} - \frac{١}{\text{ܢ}}\right)(٢\text{ܢ} + ٣) + (\text{ܢ}^{٢} + ٣\text{ܢ} + ١)\left(١ + \frac{١}{\text{ܢ}^{٢}}\right)$$

ܐܘܐ ܩܛܝܥܬܐ

$$\frac{\text{ܡܫܘܚܬܐ ܕܦܠܓܘܬܐ ܕܡܬܦܠܓܐ ܕܬܪܬܝܢ ܝܬܝܪ... } \times \text{ ... } - \text{ ... }}{(\text{ܐܘܐ ܕܡܫܬܬܐ})^{٢}}$$

ܗܢܐ ܘܕܝܢ ܐܢ ܝܗܘܒ

$\text{ܕ}(\text{ܣ}) = \frac{\text{ܗ}(\text{ܣ})}{\text{ܠ}(\text{ܣ})}$ ܟܕ $\text{ܠ}(\text{ܣ}) \neq$ ܣܘܦܪܐ

ܒܗܘܢܐ $\text{ܕ}'(\text{ܣ}) = \frac{\text{ܠ}(\text{ܣ}) \times \text{ܗ}'(\text{ܣ}) - \text{ܗ}(\text{ܣ}) \times \text{ܠ}'(\text{ܣ})}{[\text{ܠ}(\text{ܣ})]^{٢}}$

ܡܬܠܐ :

(١) $\text{ܕ}(\text{ܣ}) = \frac{٣\text{ܣ} - ١}{\text{ܣ}^{٢} + ٢}$

$$\text{ܕ}'(\text{ܣ}) = \frac{٣(\text{ܣ}^{٢} + ٢) - (٣\text{ܣ} - ١) \times ٢\text{ܣ}}{(\text{ܣ}^{٢} + ٢)^{٢}}$$

$$= \frac{٣\text{ܣ}^{٢} + ٦ - ٦\text{ܣ}^{٢} + ٢\text{ܣ}}{(\text{ܣ}^{٢} + ٢)^{٢}} = \frac{-٣\text{ܣ}^{٢} + ٢\text{ܣ} + ٦}{(\text{ܣ}^{٢} + ٢)^{٢}}$$

$$(٢)\quad \text{ܬ}(\text{ܥ}) = \frac{\text{ܥ}^{٢} - ٣\text{ܥ} + ١}{\text{ܥ}^{٢} + \text{ܥ} + ٣}$$

$$\text{ܬ}'(\text{ܥ}) = \frac{(\text{ܥ}^{٢} + \text{ܥ} + ٣)(٢\text{ܥ} - ٣) - (\text{ܥ}^{٢} - ٣\text{ܥ} + ١)(٢\text{ܥ} + ١)}{(\text{ܥ}^{٢} + \text{ܥ} + ٣)^{٢}}$$

$$(٣)\quad \text{ܨ} = \text{ܣ}^{٣} - ٢\text{ܣ} + \frac{٣}{\text{ܣ}^{٢} + ١}$$

$$\frac{\text{ܕܨ}}{\text{ܕܣ}} = \text{ܨ}' = ٣\text{ܣ}^{٢} - ٢ + \frac{\text{ܣܦܪܐ} \times (\text{ܣ}^{٢} + ١) - ٣ \times ٢\text{ܣ}}{(\text{ܣ}^{٢} + ١)^{٢}}$$

$$\frac{\text{ܕܨ}}{\text{ܕܣ}} = ٣\text{ܣ}^{٢} - ٢ - \frac{٦\text{ܣ}}{(\text{ܣ}^{٢} + ١)^{٢}}$$

ܐܣܐ ܚܫܒ̈ܢܐ :

ܐܢ ܠ ܝܗܘܒ ܗ̈ܘܐ ܡܚܠܠܬܐ ܠܣܬܠܬܐ

ܘܝܗܘܒ $\text{ܕ}(\text{ܣ}) = [\text{ܠ}(\text{ܣ})]^{\text{ܢ}}$ ܒܡܘܢܐ

$$\text{ܕ}'(\text{ܣ}) = \text{ܢ}[\text{ܠ}(\text{ܣ})]^{\text{ܢ}-١} \times \text{ܠ}'(\text{ܣ})$$

ܡܘܦܣܐ :

$$(١)\quad \text{ܕ}(\text{ܣ}) = (\text{ܣ}^{٢} - ٥\text{ܣ} + ٣)^{١٠}$$

$$\text{ܕ}'(\text{ܣ}) = ١٠(\text{ܣ}^{٢} - ٥\text{ܣ} + ٣)^{٩}(٢\text{ܣ} - ٥)$$

$$(٢)\quad \text{ܨ} = \sqrt[٣]{\text{ܣ}^{٣} - ٣\text{ܣ} + ١}$$

$$\text{ܨ} = (\text{ܣ}^{٣} - ٣\text{ܣ} + ١)^{\frac{١}{٣}}$$

$$\text{ܨ}' = \frac{١}{٣}(\text{ܣ}^{٣} - ٣\text{ܣ} + ١)^{-\frac{٢}{٣}}(٣\text{ܣ}^{٢} - ٣)$$

$$(٣)\quad \text{ܬ}(\text{ܥ}) = ٦\sqrt{٣\text{ܥ}^{٢} + ٤}$$

$$\text{ܬ}(\text{ܥ}) = ٦(٣\text{ܥ}^{٢} + ٤)^{\frac{١}{٢}}$$

$$\frac{\text{ܕܬ}}{\text{ܕܥ}} = ٣(٣\text{ܥ}^{٢} + ٤)^{-\frac{١}{٢}}(٦\text{ܥ})$$

$$= \frac{١٨\text{ܥ}}{(٣\text{ܥ}^{٢} + ٤)^{\frac{١}{٢}}} = \frac{١٨\text{ܥ}}{\sqrt{٣\text{ܥ}^{٢} + ٤}}$$

ܕܘܪܫܐ (٣ ـ ٢)

ܫ١ : ܐܫܟܚ ܫܪܒܝܬܐ (ܐܢܬܓܪܠ ܕܐܝܕܐ) ܠܟܠ ܚܕ ܡܢ ܐܠܝܢ ܕܐܬܝܢ :

(١) $\text{ܨ} = \text{ܣ}^{٢} + ٦\text{ܣ} + ٧$

(٢) $\text{ܡ}(\text{ܝ}) = (\text{ܝ}^{٣} + \text{ܝ} + ١)(\text{ܝ}^{٤} + ٣\text{ܝ}^{٢} + ٥)$

(٣) $\text{ܕ}(\text{ܣ}) = ١٠٠\left(\text{ܣ}^{٢} - \frac{\text{ܣ}^{٤}}{١٠٠٠}\right)$

(٤) $\text{ܕ}(\text{ܣ}) = \frac{٤\text{ܣ} + ١}{\text{ܣ}^{٢} - ٣\text{ܣ} - ٤}$ ، $\text{ܣ} \neq ٤$ ، $\text{ܣ} \neq -١$

(٥) $\text{ܠ}(\text{ܣ}) = \sqrt{\text{ܣ}^{٤} + ٣\text{ܣ}^{٢} + ١}$

(٦) $\text{ܟ}(\text{ܣ}) = (\text{ܣ}^{٣} - ٤)$

(٧) $\text{ܗ}(\text{ܣ}) = (\text{ܣ}^{٢} + ٥\text{ܣ} + ٧)^{\frac{٣}{٢}}$

(٨) $\text{ܕ}(\text{ܣ}) = \left(\frac{٢\text{ܣ} - ١}{\text{ܣ}^{٢} + ٣}\right)$

ܫ٢ : ܐܫܟܚ $\text{ܕ}'(\text{ܣ})$ ، $\text{ܕ}''(\text{ܣ})$ ܠܟܠ ܚܕ ܡܢ ܐܠܝܢ ܕܐܬܝܢ ܒܡܫܚܠܦܬܐ ܡܫܘܚܬܐ ܕܡܝܬܐ ܡܕܡܕܡ

(١) $\text{ܕ}(\text{ܣ}) = ٣\text{ܣ}^{٢} - ٥\text{ܣ} + ٤$ ܟܕ $\text{ܣ} = -٤$

(٢) $\text{ܕ}(\text{ܣ}) = (٢\text{ܣ} - ٥)^{٦}$ ܟܕ $\text{ܣ} = ٢$

ܦܢܝܬܐ (٣ ـ ٦) ܫܘܐܠܐ ܡܫܚܠܦܐ ܘܩܘܝܡܐ ܕܚܫܒܠܬܐ

ܡܢ ܒܬܪ ܕܝܠܦܢ ܠܟܠ ܗܢܝܢ ܫܘܐܠܐ ܡܫܚܠܦܐ ܠܫܪܒܝܬܐ ܘܫܘܐܠܐ ܩܘܝܡܐ ܠܫܪܒܝܬܐ.

ܡܕܡܠܐ ܘܕܝܕܥܢ ܕܐܝ̈ ܚܘܐ $ܨ = ܕ(ܣ)$ ܒܦܘܫܩܐ ܕܥܩܒܬܐ. $ܨ = ܕ̇(ܣ)$
ܕܦܫܩܢܐ ܠܢܛܝܬܐ ܕܡܢܓܕܢ̈ܐ ܠܝܕ ܟܠ ܢܘܩܕܬܐ ܡܢ ܢܘܩܕܘܗܝ [ܐܘ
ܦܢ̈ܝܐ ܡܢܝܬܐ ܠܢܛܝܬܐ ܕܥܩܒܬܐ]

ܡܘܦܬܐ:

ܐܫܟܚ ܫܘܝܘܬܐ ܕܡܢܓܕܢ̈ܐ ܕܥܩܒܬܐ $ܨ = ܣ^{٢} - ٥ܣ + ٢$ ܠܝܕ $ܣ = ١$

$ܨ = ܕ(ܣ) = ܣ^{٢} - ٥ܣ + ٢$

$ܨ = ܕ(١) = ١ - ٥ + ٢ = -٢$ ∴ ܢܘܩܕܬܐ ܕܠܢܝܬܐ ܗ̄ܝ $(١ ، -٢)$

$ܨ = ܕ̇(ܣ) = ٢ܣ - ٥$ [ܠܡܦܩ ܐܝܟ ܐܝܟ ܠܡܫܟܚܘܬܐ ܕܣܛܝܠܬܐ]

$ܕ̇(١) = ٢ \times ١ - ٥ = -٣$ ܢܛܝܬܐ ܕܠܢܝܬܐ ܠܝܕ $ܣ = ١$

$ܨ - ܨ_{١} = ܡ(ܣ - ܣ_{١})$

$ܨ - (-٢) = -٣(ܣ - ١)$

$ܨ + ٢ = -٣ܣ + ٣$

$٣ܣ + ܨ - ١ = ٠$ ܫܘܝܘܬܐ ܕܡܢܓܕܢ̈ܐ

ܗ̄ܝ ܘܕܡܝ ܠܡܫܟܚܘܬܐ ܕܫܘܝܘܬܐ ܕܡܢܓܕܢ̈ܐ ܠܥܩܒܬܐ ܕܐܝܬ ܝܕܝܥܬܐ ܟܡܫܟܚ ܡܕܡ̈ܐ ܬܪ̈ܝܢ ܕܢܘܩܕܬܐ ܕܠܢܝܬܐ ܡܢ ܡܫܟܚ ܢܛܝܬܐ ܟܡܫܟܚܬܐ ܕܣܛܝܠܬܐ ܘܡܫܠܡܬܐ ܠܗܘ ܬܪܘܢ̈ܐ ܗܡܝܟܢܐ ܠܢܘܩܕܬܐ ܕܠܢܝܬܐ ܘܡܦܠܚܬܐ ܕܐܫܘܝܬܐ $ܨ - ܨ_{١} = ܡ(ܣ - ܣ_{١})$

ܡܘܦܬܐ (٢):

ܐܫܟܚ ܫܘܝܘܬܐ ܕܟܠ ܡܢ ܡܢܓܕܢ̈ܐ ܘܡܩܘܡ̈ܢܐ ܥܠ ܡܢܓܕܢ̈ܐ ܕܥܩܒܬܐ

$ܨ = ܕ(ܣ) = ٢ܣ + \frac{٣}{ܣ^{٢} + ٢}$ ܠܝܕ $ܣ = -١$

ܫܪܝܐ:

ܡܫܟܚܝܢ ܡܕܡ̈ܐ ܢܘܩܕܬܐ ܕܠܢܝܬܐ

$ܨ = ܕ(ܣ) = ٢ܣ + \frac{٣}{ܣ^{٢} + ٢}$

$ܨ = ܕ(-١) = -٢ + \frac{٣}{١ + ٢} = -٢ + ١ = -١$

ܢܘܩܕܬܐ ܕܠܢܝܬܐ ܗ̄ܝ $(-١ ، -١)$

ܡܫܚܠܦܢܐ $ܨ' = ܕ'(ܗ) = ٢ \div \dfrac{٠ \times (ܗ^{٢} + ٢) - ٣\,(ܗ)}{(ܗ + ٢)}$

$ܕ'(ܗ) = ܨ' = ٢ - \dfrac{٦\,ܗ}{(ܗ^{٢} + ٢)^{٢}}$ ܣܛܝܐ ܥܠ ܥܠ ܢܘܩܙܬܐ

$ܕ'(-١) = ٢ - \dfrac{-٦}{(١ + ٢)^{٢}} = ٢ + \dfrac{٦}{٩} = ٢ + \dfrac{٢}{٣} = \dfrac{٦ + ٢}{٣}$

$= \dfrac{٨}{٣}$ ܣܛܝܐ ܥܠ ܢܘܩܙܬܐ $(-١ ، -١)$

∴ ܣܛܝܐ ܕܥܡܘܕܐ $= \dfrac{-٣}{٨}$

$ܨ - ܨ_{١} = ܡ\,(ܗ - ܗ_{١})$

ܫܘܘܝܐ ܕܡܫܝܚܝܬܐ ܗܝ :

$ܨ + ١ = \dfrac{٨}{٣}\,(ܗ + ١)$

$٣\,ܨ + ٣ = ٨\,ܗ + ٨$

$٨\,ܗ - ٣\,ܨ + ٥ = ٠$

ܫܘܘܝܐ ܕܥܡܘܕܐ ܗܝ

$ܨ + ١ = \dfrac{-٣}{٨}\,(ܗ + ١)$

$٨\,ܨ + ٨ = -٣\,ܗ - ٣$

$٣\,ܗ + ٨\,ܨ + ١١ = ٠$

ܡܬܠܐ (٣) :

ܐܫܟܚ ܢܘܩܙܬܐ ܥܠ ܥܩܘܡܬܐ ܠܥܘܩܠܐ $ܕ(ܗ) = ܗ^{٢} - ٣ܗ + ٥$

ܕܒܗ ܡܫܝܚܝܬܐ ܗܝ ܠܗ ܕܫܘܘܝܗ $ܗ - ܨ = ٥$

ܫܪܝܐ :

ܣܛܝܐ ܕܠܗܕܐ $(ܗ - ܨ = ٥) = \dfrac{ܡܩܕܡܐ\ ܗ}{ܡܩܕܡܐ\ ܨ} = ١$ ܣܛܝܐ ܕܡܫܝܚܝܬܐ

[ܒܥܘ ܡܦܝܣܢܐ // ܗܕܡܝܐ ܕܡܕܝܢܬܐ]

ܕ(ܣ) = ٢ܣ − ٣ ܓܢܝܬܐ ܕܡܦܝܣܢܘܬܐ ܓܢܝܐ ܥܠ ܢܘܩܙܬܐ

∴ ٢ܣ − ٣ = ١ ⟸ ٢ܣ = ٤ ⟸ ܣ = ٢

ܢܘܩܙܬܐ (٢ ، ٣) ܓܢܝܘ ܡܦܝܣܢܬܐ ܗܕܐ ܗܕܡܝܐ ܕܡܕܝܢܬܐ

ܡܠܒܕܢܘܬܐ :

ܡܕܡܟܐ ܘܕܡܕܟܢ ܕܐ، ܗܘܐ ܡ = ܕ(ܘ) ܦܘܫܩܐ ܐܚܪܝܢܐ

ܒܗ ܡܘܚܝܢܘܬܐ ܡ ܘܙܒܢܐ ܡܘܟܕܢܐ ܡܦܠܓܘ ، ܒܗܘܝܐ :

ܦܠܛܠܘܬܐ ܕܦܪܢܣܬܐ ܓܢܝܐ ܥܠ ܙܒܢܐ = ܦ(ܘ) = ܡ′ = ܕ′(ܘ)

ܘܫܘܕܥܬܐ ܒܥܠ ܙܒܢܐ ܫ(ܘ) = ܦ′(ܘ) = ܡ″ = ܕ″(ܘ)

ܦܘܫܩܐ (٤)

ܠܘܩܒܠ ܙܒܢ ܠܗ ܚܙܝܐ ܗܕܡܝܐ ܐܚܪܝܐ ܕܒܘܠܒܗ ܡ ܒܡܠܘܟܫܐ

ܡܦܢܕܢܐ ܒܐܚܪܢܘܬܐ ܡ(ܘ) = ܘ٢ (ܘ−٣) ܒܡܕ (ܘ) ܬܢܝܬܐ. ܐܫܟܚ

ܬܘܩܦܗ ܘܦܠܛܠܘܬܗ ܘܫܘܕܥܗ ܒܡܕ (٤) ܬܢܝܬܐ

ܡ (ܘ) = ܘ٢ (ܘ − ٣) = ܘ٣ − ٣ ܘ٢

ܦ(ܘ) = ܡ′(ܘ) = ٣ܘ٢ − ٦ܘ ܚܘܝܘܬܐ ܕܦܠܛܠܘܬܐ ܒܥܠ ܙܒܢܐ

ܫ(ܘ) = ܡ″ (ܘ) = ٦ܘ − ٦ ܚܘܝܘܬܐ ܕܫܘܕܥܬܐ ܒܥܠ ܙܒܢܐ

ܐܝܟܢ ، = ٤

ܡ = ١٦ (٤ − ٣) = ١٦ ܣܡ ܒܘܠܒܗ ܡܢ ܢܘܩܙܬܐ ܕܫܘܪܝܐ

ܦ = ٣ × ١٦ − ٦ × ٤ = ٤٨ − ٢٤ = ٢٤ ܣܡ/ܫܢ ܦܠܛܠܘܬܗ

ܐ = ٦ × ٤ − ٦ = ٢٤ − ٦ = ١٨ ܣܡ/(ܫܢܬܐ)٢ ܫܘܕܥܗ

ܦܘܫܩܐ (٥)

ܠܘܩܒܠ ܙܒܢ ܠܗ ܚܙܝܐ ܗܕܡܝܐ ܐܚܪܝܐ ܕܒܘܠܒܗ ܡ ܒܡܫܚܬܐ ܒܡܕ

(ܘ) ܡܢ ܪܩܥܐ ܝܗܒܗ ܡ(ܘ) = √(٢ ܘ٢ + ١٨) . ܐܫܟܚ ܒܘܠܒܗ ܐܝܟܢ

ܕܦܠܛܠܘܬܐ ܬܘܝܐ (١) ܡ/ܪܦ

ܡ = ܕ (ܘ) = √(٢ ܘ٢ + ١٨) = (٢ ܘ٢ + ١٨)^(١/٢)

ܦܠܛܠܘܬܐ ܦ(ܘ) = ܡ′(ܘ) = (١/٢) (٢ ܘ٢ + ١٨)^(−١/٢) (٤ ܘ)

$$= \frac{٢ܘ}{(٢ܘ^٢+١٨)^{\frac{١}{٢}}} = \frac{٢ܘ}{\sqrt{(٢ܘ^٢+١٨)}}$$

ܣܒܝܘܬܐ ܕܡܠܘܠܘܬܐ ܒܟܠ ܢܩܕܐ

ܐܢ ܡܠܘܠܘܬܐ = ١ ܡ/ܪܦ .

$$\therefore \frac{٢ܘ}{\sqrt{(٢ܘ^٢+١٨)}} = ١$$

$\therefore ٢ܘ = \sqrt{٢ܘ^٢+١٨}$ ܒܬܪܒܝܥܐ

$٤ܘ^٢ = ܘ^٢ + ١٨$

$٤ܘ^٢ - ٢ܘ^٢ = ١٨$

$٤ܘ^٢ - ٢ܘ^٢ = ١٨$

$٢ܘ^٢ = ١٨$

$ܘ^٢ = ٩$

ܗ $ܘ = ٣$ ܪܦܦܐ

ܐܘ $ܘ = -٣$ ܠܝܬ ܡܩܒܠܐ

$\therefore ܡ = ܕ(٣) = \sqrt{٢ \times ٣^٢ \times ١٨}$

$= \sqrt{١٨ + ١٨} = \sqrt{٣٦} = ٦$ ܡ ܒܘܟܪܐ

ܕܘܪܫܐ (٣ - ٣)

ܬ١ : ܐܫܟܚ ܩܘܡܝܬܐ ܕܡܫܝܚܢܐ ܕܡܠܦܐ $ܕ(ܣ) = ܣ^٣ - ٣ܣ^٢ + ٩ܣ + ٥$ ܟܕ ܢܩܘܕܐ ܣ = ܣܦܝܩܐ.

ܬ٢ : ܐܫܟܚ ܩܘܡܝܬܐ ܕܟܠ ܚܕ ܡܫܝܚܢܐ ܘܡܫܘܝܐ ܥܠ ܡܫܝܚܢܐ ܕܡܠܦܐ $ܨ = (ܣ-٣)^٣$ ܟܕ ܢܩܘܕܐ $ܣ = ٣$

ܬ٣ : ܐܫܟܚ ܢܩܘܕܬܐ ܡܬܚܡܬܐ ܠܡܠܦܐ ܕܐܝܬ $ܕ(ܣ) = ܣ^٣ - ٣ܣ^٢ - ٩ܣ + ٤$ ܐܝܟܐ ܕܗܘܐ ܡܫܝܚܢܐ ܟܢܫܐ ܡܫܘܝܐ ܠܚܕܐ ܡܫܟܚܬܐ.

ܬ٤ : ܠܡܫܚܐ ܘܝܕ ܥܠ ܣܘܟܐ ܡܬܝܪܐ ܠܦܘܬ ܐܣܝܘܬܐ $ܡ = ܕ(ܘ) = ܘ^٣ + ٣ܘ^٢ + ٤ܘ + ١$ ܟܕ ܡ ܒܡܫܚܐ ܒܡܬܪܐ ، ܘ ܙܒܢܐ ܒܦܠܓܬܐ

ܐܫܟܚ ܚܝܠܐ ܘܡܠܘܠܘܬܐ ܘܡܣܪܗܒܘܬܐ ܒܬܪ (٥) ܦܠܓܬܐ ܡܢ ܫܘܪܝܐ ܕܙܘܥܗ.

ܬ٥ : ܠܘܡܐ ܙܒܢ ܟܠ ܚܕܐ ܡܕܒܪܐ ܠܦܘܬ ܐܫܘܘܬܐ

ܐܫܟܚ . ܢܟܠܬܐ ، ܦܬܐ ܚܠܘܡܘܬܐ ١٢٠ + ٣٠ܘ − ܘ³ = ܟ(ܘ)

١) ܦܠܘܠܘܬܐ ܦܬܐ (٥) ܢܟܠܬܐ.

٢) ܒܘܟܪܗ ܐܢܝ ܕܦܬܐ ܦܠܘܠܘܬܗ ܗܟܝܢܐ.

ܬ٦ : ܐ. ܙܒܢܠܗ ܠܘܡܐ ܠܦܘܬ ܐܫܘܘܬܐ ܟ(ܘ) = ܘ³ − ٦ܘ² + ٩ܘ + ٢

ܦܕ ܟ ܒܘܟܪܐ ܒܡܟܬܒܐ ، ܙܒܢܐ ܒܕܦܦܐ . ܫܟܚ

١) ܒܘܟܪܐ ܕܠܘܡܐ ܡܢ ܒܡܘܬܐ ܕܒܘܦܢܐ ܕܙܘܟܐ ܘܡܘܪܒܗ ܐܢܝ ܕܦܬܐ ܦܠܘܠܘܬܗ ܗܟܝܢܐ.

٢) ܒܘܟܪܐ ܕܠܘܡܐ ܡܢ ܒܡܘܬܐ ܕܒܘܦܢܐ ܕܙܘܟܐ ܘܦܠܘܠܘܬܗ ܐܢܝ ܕܦܬܐ ܡܘܪܒܐ ܗܟܝܢܐ.

ܦܘܫܩ̈ܐ ܕܒܝ̈ܬܐ

ܡܦܠܚܢܘܬܐ ܥܠ ܚܫܝܠܬܐ

ܬܢܝܢܐ ܢܩܘܙܬܐ ܐܠܝܨܬܐ

> ܡܬܚܡܬܐ : ܐܢܗܘ ܗܘܐ ܕ ܐܝܬ . ܫ١ ܡܬܥܒܕܐ ܠܗ ܒܝܬ ܚܝܠܘ. ܢܩܘܙܬܐ
> (ܫ١ ، ܕ(ܫ١)) ܡܬܩܪܝܐ ܢܩܘܙܬܐ ܐܠܝܨܬܐ ⟸ ܕَ(ܫ١) = ܣܦܪܐ
> ܐܘ ܐܝܬ ܕَ ܠܐ ܡܬܝܠܐ ܚܫܝܠܬܐ ܒܗ̇ ܫ١

ܚܙܐ ܗܘܝܢ ܡܢ ܡܬܚܡܬܐ ܠܥܠ ܕܢܩܘܙܬܐ ܐܠܝܨܬܐ ܗܠܝܢ ܢܩܘܙܬܐ ܡܬܩܪܝܢܐ ܠܡܦܩܢܐ ܕܐܝܬ ܡܢ ܚܫܝܠܬܐ ܕܐܝܬ ܒܗ̇ ܗܘܝܐ ܣܦܪܐ ܐܘ ܐܝܬ ܠܐ ܡܬܝܠܐ ܚܫܝܠܬܐ ܒܗ̇ ܘ ܠܐ ܒܗ̇ ܡܬܝܠ ܐܝܟ ܬܘܕܥܐ ܕܠܘܚ ܢܩܘܙܬܐ ܐܠܝܨܬܐ ܕܗܘܝܐ ܐܝܟ ܡܬܝܠܐ ܠܚܫܝܠܬܐ ܠܗ̇ ܘܢܩܘܙܬܐ ܕܚܫܝܠܬܐ ܠܗ̇ ܣܦܪܐ ܘܠܐ ܕܐܝܬ ܠܐܝܟܐ ܐܚܪܢܐ ܡܢ ܢܩܘܙܬܐ ܐܠܝܨܬܐ.

ܦܘܫܩܐ

(١) ܐܫܟܚ ܢܩܘܙܬܐ ܐܠܝܨܬܐ ܠܐܝܬ ܕ(ܫ) = ܫ^{٣} − ٣ܫ + ٥

ܫܪܝܐ :

ܕ (ܫ) = ܫ^{٣} − ٣ ܫ + ٥

ܕَ (ܫ) = ٣ ܫ^{٢} − ٣

ܐܫܟܚ ܕَ(ܫ) = ٣ ܫ^{٢} − ٣ = ٠ ⟸ ٣ ܫ^{٢} = ٣ ⟸ ܫ^{٢} = ١

ܟܕ ܫ = ١ ، ܨ = ܕ (١) = ١ − ٣ + ٥ = ٣

ܐܘ ܫ = −١ ، ܨ = ܕ (−١) = −١ + ٣ + ٥ = ٧

∴ (١ ، ٣) ، (−١ ، ٧) ܢܩܘܙܬܐ ܐܠܝܨܬܐ

(٢) ܐܫܟܚ ܐܢ ܐܝܬ ܢܩܘܙܬܐ ܐܠܝܨܬܐ ܕܐܝܬ ܕ(ܫ) = $\frac{\text{٢ ܫ + ١}}{\text{ܫ − ١}}$ ܟܕ ܫ ≠ ١

ܫܪܝܐ :

$$\text{ܕ}(\text{ܣ}) = \frac{٢\text{ܣ} + ١}{\text{ܣ} - ١}$$

$$\dot{\text{ܕ}}(\text{ܣ}) = \frac{٢(\text{ܣ} - ١) - (٢\text{ܣ} + ١)(١)}{(\text{ܣ} - ١)} =$$

$$= \frac{٢\text{ܣ} - ٢ - ٢\text{ܣ} - ١}{(\text{ܣ} - ١)} = \frac{-٣}{(\text{ܣ} - ١)}$$

ܚܙܝܐ ܗܘܐ $\dot{\text{ܕ}}(\text{ܣ}) = \frac{-٣}{(\text{ܣ} - ١)}$ ≠ ܡܩܒܠܐ ܒܟܠ ܚܠܠܐ (−٣) ≠ ܡܩܒܠܐ

ܘܥܠ ܕܗܘ ܗܘܐ ܐܝܬܐ ܠܟܠܗ ܬܘܩܦܐ ܐܠܝܨܬܐ

ܗܢܐ ܘܒܝܢ ܠܡܫܟܚܘܬܐ ܕܬܘܩܦܐ ܐܠܝܨܬܐ ܕܐܝܬܐ ܚܕܬܐ

(١) ܡܫܟܚܝܢ $\dot{\text{ܕ}}(\text{ܣ})$

(٢) ܡܫܟܚܝܢ ܬܘܩܦܐ ܣ ܕܡܛܝܒ $\dot{\text{ܕ}}(\text{ܣ})$ = ܡܩܒܠܐ. ܗܘܝܐ ܡܫܬܘܝܐ

(٣) ܠܟܠ ܬܘܩܦܐ ܠܡܫܟܠܦ ܣ ܡܢܟܝܢ ܚܠܠܐ ܒܗ ܦܩܘܕܬܐ (٢) ܡܫܟܚܝܢ ܝ = $\text{ܕ}(\text{ܣ})$ ܒܐܝܕܐ ܫܪܝܟ ܚܠ ܬܘܩܦܐ ܐܠܝܨܬܐ.

ܦܣܩܐ (٤ . ٢) ܐܝܬ ܡܫܬܢܝܢܐ ܡܣܩܐ ܘܐܝܬܐ ܡܫܬܢܝܢܐ ܢܚܬܐ :

ܡܫܘܕܥܬܐ : ܢܩܘܡ ܗܘܐ ܕ ܐܝܬܐ ، ܥ ܒܬܘܩܦܐ ܡܬܦܬܚܬܐ ܠܐ ܡܩܒܠܬܐ
ܡܢ ܢܟܝܠܘ
ܒܗܘܐ :

(١) ܕ ܐܝܬܐ ܡܣܩܢܝܬܐ ܒܗ ܥ ⟺ ܠܟܠ $\text{ܣ}_١ \in \text{ܥ}$ ܘܠܟܠ $\text{ܣ}_٢ \in \text{ܥ}$
$(\text{ܣ}_١ > \text{ܣ}_٢ \Rightarrow \text{ܕ}(\text{ܣ}_١) > \text{ܕ}(\text{ܣ}_٢))$

ܗܢܐ ܘܒܝܢ ܒܡܐ ܕܢܝܚ ܡܫܠܘܡܝܐ ܡܣܩܢܝܐ ܢܘܪܗ ܘܒܡܐ ܕܢܚܬ ܡܫܠܘܡܝܐ ܢܚܬܐ ܢܘܪܗ

(٢) ܕ ܐܝܬܐ ܢܚܬܢܝܬܐ ܒܗ ܥ ⟺ ܠܟܠ $\text{ܣ}_١ \in \text{ܥ}$ ܘܠܟܠ $\text{ܣ}_٢ \in \text{ܥ}$
$(\text{ܣ}_١ > \text{ܣ}_٢ \Rightarrow \text{ܕ}(\text{ܣ}_١) < \text{ܕ}(\text{ܣ}_٢))$

ܗܢܐ ܘܒܝܢ ܒܡܐ ܕܢܝܚ ܡܫܠܘܡܝܐ ܢܚܬܐ ܢܘܪܗ ܘܒܡܐ ܕܢܚܬ ܡܫܠܘܡܝܐ ܡܣܩܐ ܢܘܪܗ

ܡܬܠܐ (١)

ܕ : ܚ ← ܚ   ܕ(ܣ) = ܣ²

ܢܚܙܐ ܕܦܘܢܩܨܝܐ ܕܝܗܝܒܐ ܠܥܠ ܗܘܐ ܐܝܟ ܡܚܘܝܐ ܒܨܘܪܬܐ ܕܠܬܚܬ :

ܨ

ܣ

ܝܘܬܪܐ ܕܐܝܢܚܐ ܣ

ܨܘܪܬܐ (٤ ـ ١)

ܘܡܚܘܐ ܗܘܐ ܡܢ ܨܘܪܬܐ ܠܟܠ ܣ > ܣܦܪܐ . ܟܕ ܕܝܘܬܪܐ ܬܘܣܦܬܐ ܣ ܬܘܣܦܬܐ ܕ(ܣ) ܝܬܝܪܐ ܐܦ ܗܝ :

ܗܢܐ ܝܬܝܪܬܐ ܗܘ ܥܠ ܚܘܕܪܐ ؛ ܣ : ܣ ܚ . ܣ > ܣܦܪܐ ؛ ܐܠܐ ܠܟܠ ܣ < ܣܦܪܐ ܡܚܘܐ ܗܘܐ ܕܟܕ ܝܬܝܪܐ ܬܘܣܦܬܐ ܣ ܬܘܣܦܬܐ ܕ(ܣ) ܒܨܝܪܐ ܐܦ ܗܝ :

ܗܢܐ ܒܨܝܪܬܐ ܥܠ ܚܘܕܪܐ ؛ ܣ : ܣ ܚ . ܣ < ܣܦܪܐ ؛ ܘܡܫܘܕܥܐ ܗܘ ܡܬܚܙܝܐ ܕܦܢܝܬܐ ܕܝܘܬܪܐ ܐܘ ܒܨܝܪܘܬܐ ܕܦܘܢܩܨܝܐ ܡܬܝܕܥܐ ܠܡܬܠܬܐ ܒܥܘܕܪܢܐ ܕܫܘܚܠܦܐ ܕܩܕܡܝܐ ܕܡܫܬܠܡܬܐ . ܡܫܘܕܥܢܘܬܐ ܕܐܬܐ ܠܬܚܬ ܠܗܕܐ.

ܡܫܘܕܥܢܘܬܐ :

ܐܢ ܗܘܬ ܗܢܐ ܕ ܦܘܢܩܨܝܐ ܠܡܬܠܬܐ ܥܠ ܚܘܕܪܐ (ܐ ، ܒ) ܒܥܘܕܪܢܐ :

(١) ܕ ܝܬܝܪܬܐ ܥܠ (ܐ ، ܒ)   ⟺   ܕ′(ܣ) > ܣܦܪܐ ܠܟܠ ܣ ∈ (ܐ ، ܒ)

(٢) ܕ ܒܨܝܪܬܐ ܥܠ (ܐ ، ܒ)   ⟺   ܕ′(ܣ) < ܣܦܪܐ ܠܟܠ ܣ ∈ (ܐ ، ܒ)

ܡܬܠܐ :

ܠܟܠ ܚܕ ܡܢ ܗܠܝܢ ܕܠܬܚܬ ܐܫܟܚ ܐܝܟ ܚܘܕܪܐ ܕܦܘܢܩܨܝܐ . ܦܢܝܬܐ ܕܝܘܬܪܐ . ܦܢܝܬܐ ܕܒܨܝܪܘܬܐ ܕܗܕܐ .

(١) ܕ(ܣ) = ܣ² − ٤ ܣ + ٣

ܡܬܥܒܕܐ ܕ′(ܣ) = ٢ܣ − ٤

ܡܫܘܚ ܕ(ܣ) = ٢ܣ − ٤ = ٠ ⟸ ٢ܣ = ٤ ⟸ ܣ = ٢

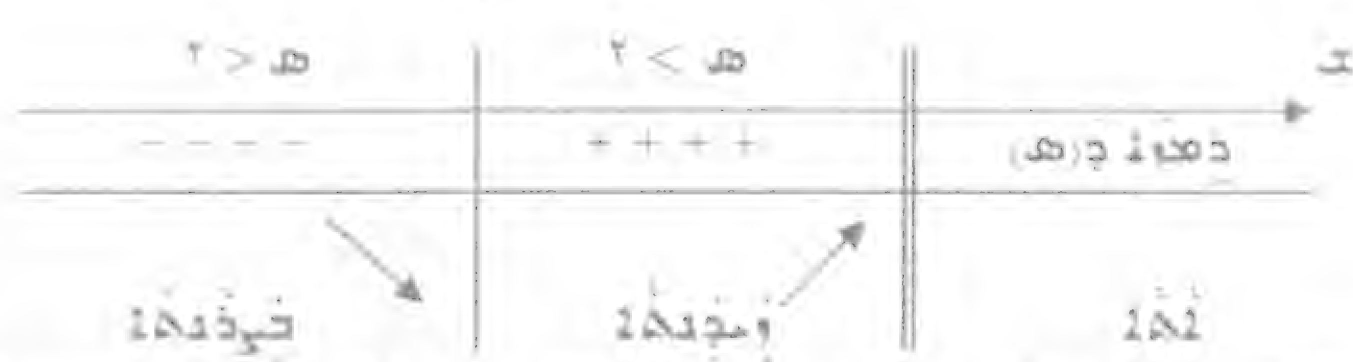
∴ ܐܝܬ ܢܘܩܙܬܐ ܐܠܝܬܐ ܠܘܬ ܣ = ٢ ܡܚܫܒ ܡܘܦܩܢܐ ܨ̈ܒܝܐ

ܨ = ܕ (٢) = ٤ − ٨ + ٣ − ١

(٢ ، −١) ܢܘܩܙܬܐ ܐܠܝܬܐ

ܘܠܡܫܟܚܘܬܐ ܕܦܣܩܬܐ ܕܙܘܕܝܬܐ ܐܘ ܒܨܝܪܘܬܐ ܡܫܚܠܦ ܕܡܘܬ ܕ(ܣ) ܘܗܘܐ ܒܡܬܚܐ ܒܣܝܩܐ ܕܡܢܝܢܐ ܫܪܝܪܐ ܒܐܘܪܚܐ ܕܐܬܝܐ :

ܕܒܨܝܪ ܣܝܩܐ ܕܡܢܝܢܐ ܘܡܫܚܠܦ ܟܠܗ ܒܢܘܩܙܬܐ ܣ ܕܠܘܬܗ̇ ܢܘܩܙܬܐ ܐܠܝܬܐ ܘܠܘܬܗ̇ ܦܠܓ ܣܝܩܐ ܕܡܢܝܢܐ ܠܬܪܬܝܢ ܦܣܩܬܐ ܡܢ ܡܬܚܐ ܐܝܟ ܒܐܣܟܡܐ ܠܬܚܬ .



ܡܢ ܡܠܟܫ ܡܬܚܙܝܐ ܕܗ ܥܠ ܒܢܝܢܐ ܘܡܫܚܠܦܢܘܬܐ ܕܗ ܕ(ܣ) ܘܡܫܘܚܬܐ ܕܡܘܬ ܕܡܫܘܚܬܐ ܒܫܘܝܐ ܥܠ ܕܡܘܬ ܕ(ܣ) ܒܗܝ ܒܢܝܢܐ ܕܡܬܚܙܝܐ ܗܘ ܡܬܚܙܝܢܘܬܐ ܒܐܝܕܐ ܦܣܩܬܐ ܡܫܚܠܦ ܥܠ ܕܡܘܬܐ ܐܝܟ ܡܢ (٢) ܡܢ ܦܣܩܬܐ ܣ = ٣ ܘܚܙܝ ܠܕܡܘܬ ܕ(٣) ܕܗܝ ܗܝ ܕܡܫܚܠܦܐ ܕܐ ܡܢ ܕ(ܣ) < ܡܦܩܢܐ ܠܟܠ ܣ > ٢ ، ܘܗܟܢ ܡܬܚܙܝܐ ܡܫܚܠܦ ܘܡܢܐ ܡܢ (٢) ܡܢ ܦܣܩܬܐ ܣ = ١ ܘܚܙܝ ܠܕܡܘܬ ܕ(١) ܕܗܝ ܗܝ ܕܐ ܡܢ ܕ(ܣ) > ܡܦܩܢܐ ܠܟܠ ܣ < ٢ ܘܡܢ ܕܐܝܬ ܒܗܘܐ

ܐܦܐ ܕܙܘܕܝܬܐ ܕܗ (١) { ܣ : ܣ ∈ ܚ ، ܣ < ٢ }

ܘܒܨܝܪܘܬܐ ܕܗ (٢) { ܣ : ܣ ∈ ܚ ، ܣ > ٢ }

ܡܬܚܙܝܢܘܬܐ (٢) :

ܕ (ܣ) = ܣ^{٣} − ٣ ܣ + ٢

ܐܫܟܚ ܢܘܩܙܬܐ ܐܠܝܬܐ ܘܬܚܬܝܬܐ :

ܕ′ (ܣ) = ٣ ܣ^{٢} − ٣ = ٠ [ܒܡܫܘܚܬܐ ܕ′(ܣ) = ܡܦܩܢܐ]

٣ ܣ^{٢} = ٣ ⟸ ܣ^{٢} = ١

ܟܕ ܣ = ١ ، ܨ = ܕ(١) = ܣܦܝܪܐ

ܘܐ ܣ = −١ ، ܨ = ܕ(−١) = −١ + ٣ + ٢ = ٤

ܐܝܟ ܢܘܩܙܬܐ ܐܠܝܨܬܐ ܠܦܬ ܣ = ١ ; ܣ = −١ ܝܗܠܗ ܢܘܩܙܬܐ (١ ، ٠) ، (−١ ، ٤)

ܠܡܢܝܢܬܐ ܕܦܠܓܬܐ ܕܡܢܝܢܬܐ ܘܫܪܝܪܬܐ ܠܬܠܬ ܦܣܩܬܐ ܚܕܐ ܕܡܢܝܢܐ ܟܪܝܟܐ

ܘܦܣܩܝܢ ܬܠܬܐ

ܣ = ١ ، ܣ = −١

ܒܪܐ ܐܘܦܩܐ ܦܣܩܝܢ ܕܫܘܐ ܕ(ܣ) ܒܗ ܦܣܩܬܐ ܕܐܬܝ :

{ܣ ∈ ܚ ، ܣ > ١} ، ܦܣܩܐ (−١ ، ١) ، {ܣ ∈ ܚ ، ܣ < −١}

ܚ	١ < ܣ	ܣ	ܣ < −١
		١	−١
ܫܘܐ ܕ(ܣ)	+ + +	− − − −	+ + + +
ܕ(ܣ)	↗	↘	↗

ܘܐܬܐ ܒܡܫܠܡܢܘܬܐ ܡܬܦܫܛܐ ܒܗ ܦܣܩܬܐ ܡܚܡܬܐ ܐܝܟ ܣ = ٢ ܡܫܚܠܦܝܢ ܕܫܘܐ ܕ(ܣ) ܘܫܘܝܢ ܕܝܠܗ ܡܫܘܚܬܐ ܗܘܐ ܘܟܡ ܕ(ܣ) > ܣܦܝܪܐ ܠܟܠ ܣ > ١ . ܟܕ ܡܢܚܬ ܡܬܦܫܛܐ ܒܗ ܦܣܩܐ ܦܬܝܚܐ (−١ ، ١) ܐܝܟ ܣ = ܣܦܝܪܐ ܘܡܫܚܠܦܝܢ ܕܫܘܐ ܕ(ܣ) ܣܦܝܪܐ ، ܘܫܘܝܢ ܕܝܠܗ ܒܠܚܘܕ ܗܘܐ ܘܟܡ ܕ(ܣ) < ܣܦܝܪܐ ܠܟܠ ܣ ∈ (−١ ، ١) ܘܡܘܒܠܐ ܘܡܬܦܫܛܐ ܒܗ ܟܠ ܦܣܩܬܐ ܘܦܣܩܝܢ ܕܫܘܐ ܕܚܝܠܬܐ ܡܫܡܫܬܐ ܘܡܢܗ ܦܣܩܝܢ ܘܚܕܬܐ ܐܘ ܬܪܝܢܬܐ ܕܐܬܐ ܐܬܐ ܒܗܝ ܦܣܩܬܐ ܘܡܫܝܚܬܐ ܝܠܗ ܠܚܕܬܐ ܕܐܘܐ ܦܠܘܢܐ ܡܫܬܢܝܐ ܐܘ ܟܠ ܒܢܝܢܐ ܐܣܪܢܐ ܡܢ ܬܪܬܝܢ ܘܗܘܝܐ :

ܐܬܐ ܕ ܙܘܕܬܐ ܝܠܗ ܒܗ (١) {ܣ : ܣ ∈ ܚ ، ܣ > ١}

(٢) {ܣ : ܣ ∈ ܚ ، ܣ < −١}

ܘܐܬܐ ܬܪܝܬܐ ܝܠܗ ܒܗ ܦܣܩܐ ܦܬܝܚܐ (−١ ، ١)

ܦܠܘܚܬܐ (٣)

ܕ(ܣ) = (٢ − ܣ)^{٣}

ܕ̄(ܣ) = ٣ (٢ − ܣ)^{٢} (−١) = −٣ (٢ − ܣ)^{٢}

ܢܚܒܘܫ : $-٣ (٢ - ܗ)^{٢} = ٠ \Leftarrow$

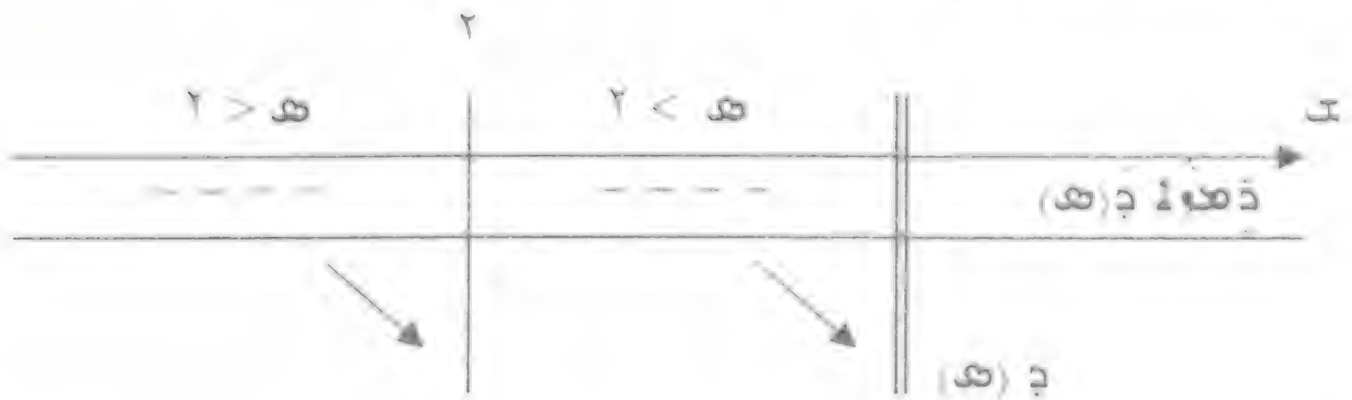
$(٢ - ܗ)^{٣} = ٠ \Leftarrow ٢ - ܗ =$ ܨܦܪܐ

ܗ = ٢ ܐܝܟ ܢܘܩܙܬܐ ܐܠܝܨܬܐ ܠܘܬ ܗ = ٢

$ܕ(٢) = (٢ - ٢)^{٣} =$ ܨܦܪܐ

(٢ ، ٠) ܢܘܩܙܬܐ ܐܠܝܨܬܐ

ܘܒܕܒܩܬܐ ܕܣܝܡܐ ܕܦܫܝܛܐ ܘܡܫܚܬܐ ܕܪܦܘܐ ܕ(ܗ) ܐܝܟ ܡܬܚܙܝܐ ܒܥܠܡܐ ܒܚܙܘ ܠܗܘܢܬܐ ܠܬܚܬ.



ܚܙܝ ܗܘܐ ܕ :

ܐܬܐ ܕ ܡܬܪܒܝܬܐ ܡܠܘ ܒܗ (١) {ܗ : ∋ ܚ ، ܗ > ٢}

(٢) {ܗ : ∋ ܚ ، ܗ < ٢}

ܘܠܝܬ ܦܢܝܬܐ ܕܘܡܝܬܐ.

ܦܢܝܬܐ (٤ ـ ٣) ܥܘܠܡܐ ܪܒܐ ܘܙܥܘܪܐ ܕܘܟܬܢܝܐ

ܢܣܒܘܢ ܗܘܝܐ ܨ = ܕ(ܗ) ܦܘܫܩܢܬܐ ܕܝܕܝܥܬܐ ܒܐܣܟܡܐ ܕܐܬܐ ܘܦܬܟܪ ܒܕܘܟܝܬܐ ܒܢܘܢ ܕܦܢܝܐ ܘܚܙܝ ܡܢ ܕܐܬܐ :



ܐܣܟܡܐ (٤ ـ ٢)

١) ܚܘܫܒܐ ܕܐܪܐ ܕܡܢܝܢܐ ܗܠܝܢ ܘܟܠܗ ܡܬܩܪܝܐ ܙܘܘܝܬܐ. ܘܟܠ ܚܕ ܡܢ ܗܠܝܢ ܙܘܘܝܬܐ
(ܕ(ܐ) ، (ܘ ، ܕ(ܘ)) ܐܠܝܦܬܐ ܗܠܝܢ ܗܘ ܡܩܒܠܢܐ ܠܗܕܐ ܗܘܐ ܡܫܐ
ܡܫܘܚܬܐ ܗܢܐ ܘܡܢ ܙܘܘܝܬܐ ܕܡܬܩܪܝܐ ܠܗܕܐ = ܡܫܘܚܬܐ.

٢) ܕ(ܐ) ܐܢ ܗܠܝܢ ܕܝ ܙܘܘܝܬܐ ܠܐܚܪܢܐ ܠܗ ܠܡܬܚܬ ܐܝܟ ܐܢܐ ܙܘܘܝܬܐ ܕܝ ܡܢܗ
ܐܝܟ ܠܟܠ ܗ ؛ ܕܘܪܐ ܦܫܝܛܐ (ܠ ، ܒ) ܒܗܘܐ ܕ(ܗ) ≥ ܕ(ܐ) ܗܢܐ ܘܡܢ ܕ(ܐ)
ܕܝ ܙܘܘܝܬܐ ܠܐܚܪܢܐ ܠܗ ܕܘܪܐ ܦܫܝܛܐ (ܐ ، ܒ) ܡܬܚܬܝܐ ܠܗ ܠܡܬܚܬ ܕܐܚܪܐ
ܐܝܟ ܕܐܝܬ ܙܘܘܝܬܐ ܦܬܝܬܐ ܡܕܡܐ ܕܝ ܙܘܘܝܬܐ ܕܡܬܚܬܝܐ (ܬܘܠܡܐ ܕܝܐ
ܕܡܬܚܬܝܐ) ܠܐܚܪܢܐ ܘܡܘܡܬܐ (ܐ ، ܕ(ܐ)) ܦܬܝܬܐ ܡܕܡܐ ܬܘܠܡܐ ܕܝܐ
ܕܡܬܚܬܝܐ.

٣) ܘܐܗܐ ܗܘܟܢܐ ܗܠܝܢ ܕ(ܗ) > ܡܩܒܠܐ ܠܟܠ ܗ ؛ (ܐ ، ܐ) ܗܢܐ ܘܡܢ ܐܢܐ
ܘܡܕܡܐ ܗܠܝܢ ܡܕܡ ،) ܘ ܕ(ܗ) > ܡܩܒܠܐ ܠܟܠ ܗ ؛ (ܐ ، ܒ) (ܗܢܐ ܘܡܢ ܐܢܐ
ܚܪܝܦܬܐ ܗܠܝܢ ܒܗܕ ܐ).

٤) ܕ(ܘ) ܐܢ ܗܠܝܢ ܘܚܕ ܙܘܘܝܬܐ ܠܐܚܪܢܐ ܠܗ ܠܡܬܚܬ ܐܝܟ ܐܢܐ ܙܘܘܝܬܐ
ܘܚܕ ܡܢܗ ܐܝܟ ܠܟܠ ܗ ؛ ܕܘܪܐ ܦܫܝܛܐ (ܠ ، ܗ) ܒܗܘܐ ܕ(ܗ) ≥ ܕ(ܘ) ܗܢܐ
ܘܡܢ ܕ(ܘ) ܘܚܕ ܙܘܘܝܬܐ ܠܐܚܪܢܐ ܠܗ ܕܘܪܐ (ܠ ، ܗ) ܡܬܚܬܝܐ ܠܗ ܠܡܬܚܬ
ܐܢܐ ܐܝܟ ܕܐܝܬ ܙܘܘܝܬܐ ܦܬܝܬܐ ܡܕܡܐ ܘܚܕ ܙܘܘܝܬܐ ܕܡܬܚܬܝܐ
ܬܘܠܡܐ ܘܚܕܐ ܕܡܬܚܬܝܐ) ܘܡܘܡܬܐ (ܘ ، ܕ(ܘ)) ܦܬܝܬܐ ܡܕܡܐ ܙܘܘܝܬܐ
ܕܬܘܠܡܐ ܘܚܕܐ ܕܡܬܚܬܝܐ.

٥) ܘܐܗܐ ܗܘܟܢܐ ܗܠܝܢ ܕ(ܗ) > ܡܩܒܠܐ ܠܟܠ ܗ ؛ (ܠ ، ܘ) (ܗܢܐ ܘܡܢ ܐܢܐ
ܚܪܝܦܬܐ ܗܠܝܢ ܡܕܡ ܘ) ܘ ܕ(ܗ) > ܡܩܒܠܐ ܠܟܠ ܗ ؛ (ܘ ، ܗ) (ܗܢܐ ܘܡܢ ܐܢܐ
ܘܡܕܡܐ ܒܗܕ ܘ).

ܡܫܠܡܢܘܬܐ :

ܟܠ ܙܘܘܝܬܐ ܕܬܘܠܡܐ ܕܝܐ ܕܡܬܚܬܝܐ ܐܘ ܙܘܘܝܬܐ ܕܬܘܠܡܐ ܘܚܕܐ
ܕܡܬܚܬܝܐ ܗܠܝܢ ܙܘܘܝܬܐ ܐܠܝܦܬܐ ܐܝܟ ܙܘܘܝܬܐ ܐܠܝܦܬܐ ܐܢ ܗܠܝܢ ܬܕܝܪܐ
ܕܗܘܐ ܙܘܘܝܬܐ ܕܬܘܠܡܐ ܕܝܐ ܕܡܬܚܬܝܐ ܐܘ ܙܘܘܝܬܐ ܕܬܘܠܡܐ ܘܚܕܐ
ܕܡܬܚܬܝܐ. ܘܡܬܘܡܬܐ ܗܘܝܢ ܡܢ ܗܕ ܒܬܕܢܗ ܕܒܝܕ ܕܫܡܫ ܠܗ ܒܗܕܡ
ܐܝܟ ܕܐܝܬ ܙܘܘܝܬܐ ܐܠܝܦܬܐ ܕܗܠܝܢ ܡܢ ܐܕܝܐ ܕܠܗܕܐ ܐܝܟ ܐܢܐ ܠܟ ܡܚܠܐ
ܚܠܝܬܐ.

ܡܥܒܕܢܘܬܐ :

ܡܢ ܡܐ ܕܥܒܕܠܗ ܦܪܥܐ ܗܘܝ ܡܐ ܡܫܟܚܬܐ ܕܢܘܩܙܐ ܕܥܘܠܝܐ ܪܒܐ ܘܙܥܘܪܐ ܕܕܘܟܢܝܐ ܡܚܫܚܝ ܠܦܫܘܛܘܬܐ ܕܐܬܐ :

(١) ܡܫܟܚܝܢ ܢܘܩܙܐ ܐܠܝܨܬܐ ܐ، ܐܝܟ ܐܢܝ ܕܡܥܒܕܠܗ ܒܨܦܪ (ܐ، ܐܬܐ ܠܐܬܠܗ ܢܘܩܙܐ ܐܠܝܨܬܐ ܒܗܘܐ ܠܗ ܢܘܩܙܐ ܕܥܘܠܝܐ ܪܒܐ ܕܕܘܟܢܝܐ ܐܘ ܢܘܩܙܐ ܕܥܘܠܝܐ ܘܙܥܘܪܐ ܕܕܘܟܢܝܐ).

(٢) ܡܫܚܝ ܦܢܝܬܐ ܕܡܢܝܬܐ ܕܐܬܐ ܘܦܢܝܬܐ ܕܣܝܡܬܗ ܐ، ܐܝܟ

(٣) ܐ، ܐܬܐ ܝܗܘܐ ܡܢܝܬܐ (ܗܘܐ ܘܕܝܢ ܡܥܘܐ ܕܡܫܟܠܬܐ ܡܕܡܝܬܐ ܕܐܬܐ ܐܬܐ ܫܘܡܠܐ) ܡܢ ܢܘܩܙܐ ܐܠܝܨܬܐ ܘܣܝܡܬܐ ܒܬܪܗ (ܗܘܐ ܘܕܝܢ ܡܥܘܐ ܕܡܫܟܠܬܐ ܡܕܡܝܬܐ ܕܐܬܐ ܠܠܦܘܬ ܒܬܪ ܢܘܩܙܐ ܐܠܝܨܬܐ)، ܢܘܩܙܐ ܐܠܝܨܬܐ ܗܕܝܢ ܝܠܗ ܢܘܩܙܐ ܕܥܘܠܝܐ ܪܒܐ ܕܕܘܟܢܝܐ.

(٤) ܘ، ܐܬܐ ܝܗܘܐ ܣܝܡܬܐ (ܗܘܐ ܘܕܝܢ ܡܥܘܐ ܕܡܫܟܠܬܐ ܡܕܡܝܬܐ ܕܐܬܐ ܠܠܦܘܬ) ܡܢ ܢܘܩܙܐ ܐܠܝܨܬܐ ܘܡܢܝܬܐ ܒܬܪܗ (ܗܘܐ ܘܕܝܢ ܡܥܘܐ ܕܡܫܟܠܬܐ ܠܬܐ ܫܘܡܠܐ ܒܬܪ ܢܘܩܙܐ ܐܠܝܨܬܐ)، ܢܘܩܙܐ ܐܠܝܨܬܐ ܗܕܝܢ ܝܠܗ ܢܘܩܙܐ ܕܥܘܠܝܐ ܘܙܥܘܪܐ ܕܕܘܟܢܝܐ.

(٥) ܐ، ܠܐ ܒܪܐܠܗ ܒܫܘܠܦܐ ܝܠܗ ܡܥܘܐ ܕ(ܣ) ܡܫܚܠܦܬܐ ܒܢܘܩܙܐ ܐܠܝܨܬܐ ܗܕܝܢ ܐܬܐ ܠܐܬܠܗ ܢܘܩܙܐ ܕܥܘܠܝܐ ܪܒܐ ܕܕܘܟܢܝܐ ܐܘ ܢܘܩܙܐ ܕܥܘܠܝܐ ܘܙܥܘܪܐ ܕܕܘܟܢܝܐ ܘܢܘܩܙܐ ܐܠܝܨܬܐ ܗܘܝܐ ܒܫܘܡ ܢܘܩܙܐ ܐܠܝܨܬܐ.

ܡܬܠܐ :

ܠܟܠ ܡܢ ܐܬܐ ܕܐܬܐ ܐܫܟܚ ܐ، ܐܝܟ ܢܘܩܙܐ ܕܥܘܠܝܐ ܪܒܐ ܘܙܥܘܪܐ ܕܕܘܟܢܝܐ :

(١) ܕ (ܣ) = $ܣ^{٣}$ − ٣ $ܣ^{٢}$ − ٩ ܣ + ٧

ܕ′ (ܣ) = ٣ $ܣ^{٢}$ − ٦ ܣ − ٩

ܕ′ (ܣ) = ٣ $ܣ^{٢}$ − ٦ ܣ − ٩ = ܨܦܪܐ

∴ $ܣ^{٢}$ − ٢ ܣ − ٣ = ٠

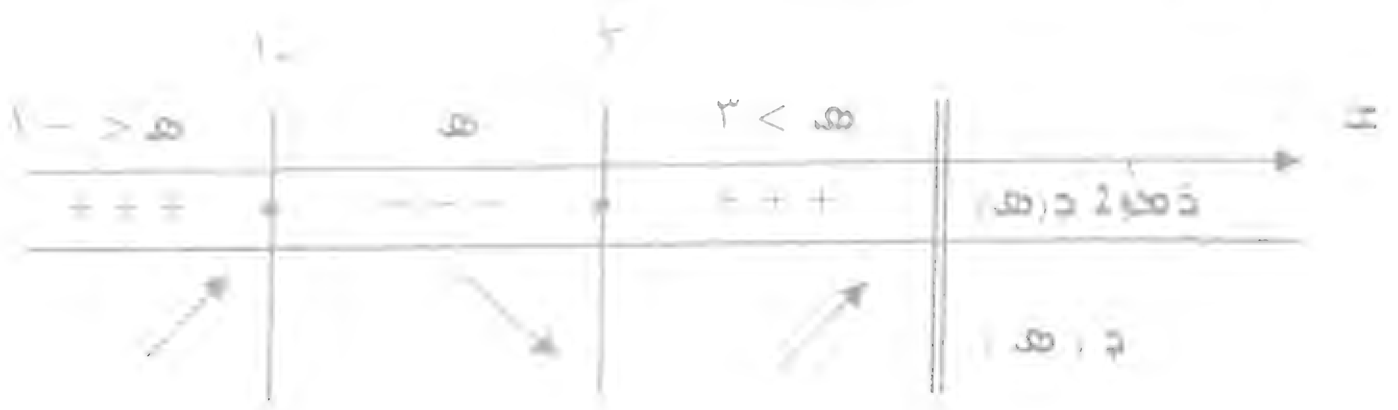
(ܣ − ٣) (ܣ + ١) = ܨܦܪܐ

ܐܢ ܣ = ٣ ، ܨ = ܕ (٣) = ٢٧ − ٢٧ − ٢٧ + ٧ = − ٢٠

ܐܘ ܣ = −١ ، ܨ = ܕ (−١) = −١ − ٣ × ١ + ٩ + ٧ = ١٢

ܢܘܩܙܐ ܐܠܝܨܬܐ (١٢ ، −١) ، (−٢٠ ، ٣)

ܕ̇ (ܗ) = ٣ ܗ^{٢} − ٦ ܗ − ٩



ܐܬܐ ܘܡܩܒܠܬܐ ܡܠܘ ܠܗ (١) {ܗ : ܗ ∈ ܚ ، ܗ > ٣}

(٢) {ܗ : ܗ ∈ ܚ ، ܗ < −١}

ܘܡܚܪܬܐ ܠܗ ܒܘܚܐ (−١ ، ٣)

(٣ ، −٢٠) ܢܘܩܙܬܐ ܕܥܘܠܡܐ ܘܠܘܬܐ ܕܕܠܝܠ̈ܬܐ

(−١ ، ١٢) ܢܘܩܙܬܐ ܕܥܘܠܡܐ ܕܬܚܬܐ ܕܕܠܝܠ̈ܬܐ

(٦) ܕ (ܗ) = ܗ^{٤} − ٢ ܗ^{٢} + ١

ܕ̇ (ܗ) = ٤ ܗ^{٣} − ٤ ܗ

ܕ̇ (ܗ) = ٤ ܗ^{٣} − ٤ ܗ = ܣܦܝܩܬܐ

٤ ܗ (ܗ^{٢} − ١) = ٠

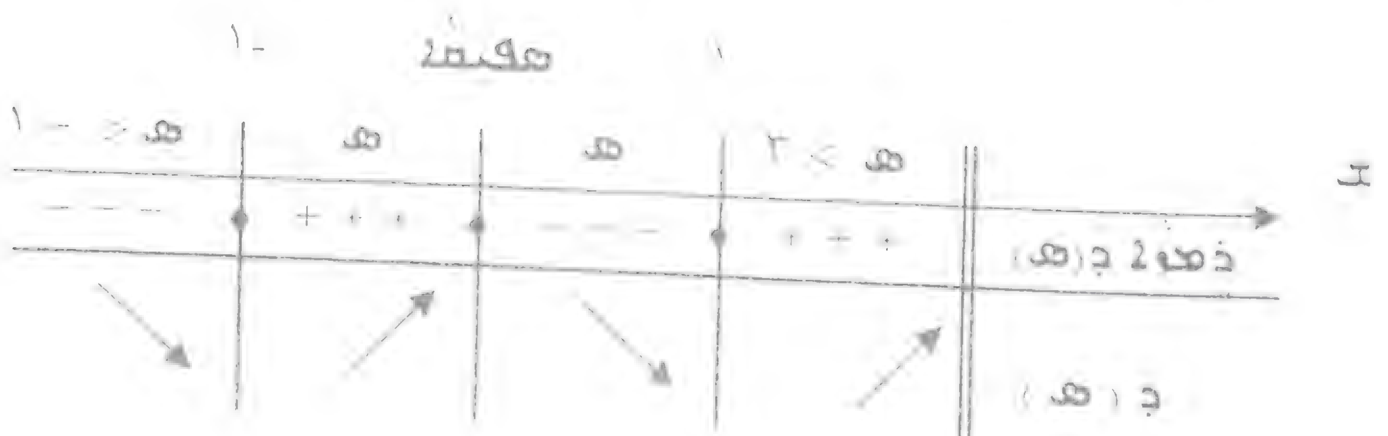
٤ ܗ (ܗ − ١) (ܗ + ١) = ܣܦܝܩܬܐ

ܡ ܗ = ٠ ، ܨ = ܕ (٠) = ١

ܐܘ ܗ = ١ ، ܨ = ܕ (١) = ܣܦܝܩܬܐ

ܐܘ ܗ = −١ ، ܨ = ܕ (−١) = ܣܦܝܩܬܐ

(٠ ، ١) ، (١ ، ٠) ، (−١ ، ܣܦܝܩܬܐ) ܐܠܝܢ̈ܐ



ܐܵܗܵܐ ܡܬܪܲܒܝܵܢܬܵܐ ܥܲܠ

(١) $\{\text{ܣ} : \text{ܣ} \in \text{ܚ} ، \text{ܣ} > ١\}$ (٢) ܦܘܫܟܐ $(-١ ، ٠)$

ܘܡܙܘܥܪܢܬܐ ܥܲܠ

(١) $\{\text{ܣ} : \text{ܣ} \in \text{ܚ} ، \text{ܣ} < -١\}$ (٢) ܦܘܫܟܐ $(٠ ، ١)$

∴ ܢܘܩܙܐ $(١ ، ٠)$ ، $(-١ ، ٠)$ ܗܠܝܢ ܢܘܩܙܐ ܕܟܘܠܡܐ ܙܥܘܪܐ ܕܘܟܬܢܝܐ

ܘܢܘܩܙܬܐ $(٠ ، ١)$ ܢܘܩܙܬܐ ܕܟܘܠܡܐ ܪܒܐ ܕܘܟܬܢܝܐ

ܡܘܦܩܬܐ (٣) :

$$\text{ܬ}(\text{ܣ}) = \text{ܣ}^{٣}(-٤ + \text{ܣ})$$

$$\text{ܬ}(\text{ܣ}) = -٤\text{ܣ}^{٣} + \text{ܣ}^{٤}$$

$$\text{ܬ}'(\text{ܣ}) = -١٢\text{ܣ}^{٢} + ٤\text{ܣ}^{٣}$$

$$\text{ܬ}'(\text{ܣ}) = -١٢\text{ܣ}^{٢} + ٤\text{ܣ}^{٣} = ٠$$

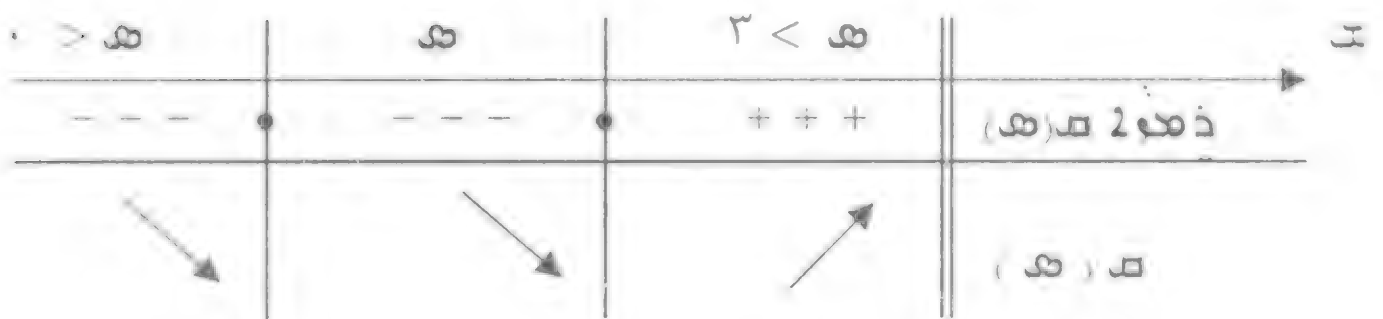
$$٤\text{ܣ}^{٢}(-٣ + \text{ܣ}) = ٠$$

ܐܢ $٤\text{ܣ}^{٢} = ٠ \Rightarrow \text{ܣ} = ٠ ، \text{ܨ} = \text{ܕ}(٠) = $ ܡܦܝܩܬܐ

ܐܘ $\text{ܣ} = ٣ \Rightarrow \text{ܨ} = \text{ܕ}(٣) = -٢٧$

∴ $(٠ ، ٠)$ ، $(٣ ، -٢٧)$ ܢܘܩܙܬܐ ܐܠܟܣܝܬܐ

٣



ܚ	$\text{ܣ} > ٣$	ܣ	$\text{ܣ} < ٠$
ܪܘܫܐ $\text{ܬ}'(\text{ܣ})$	+ + +	− − −	− − −
$\text{ܬ}(\text{ܣ})$	↗	↘	↘

ܐܵܗܵܐ ܡܬܪܲܒܝܵܢܬܵܐ ܗܠܝܢ ܥܲܠ ܣ : $\{\text{ܣ} \in \text{ܚ} ، \text{ܣ} > ٢\}$

ܘܡܙܘܥܪܢܬܐ ܥܲܠ (١) ܦܘܫܟܐ $(٠ ، ٣)$

(٢) $\{\text{ܣ} : \text{ܣ} \in \text{ܚ} ، \text{ܣ} <$ ܡܦܝܩܬܐ$\}$

$(٣ ، -٢٧)$ ܢܘܩܙܬܐ ܕܟܘܠܡܐ ܙܥܘܪܐ ܕܘܟܬܢܝܐ

ܢܘܩܙܬܐ $(٠ ، ٠)$ ܗܠܝܢ ܟܠܢܘܟ ܢܘܩܙܬܐ ܐܠܟܣܝܬܐ ܠܐ ܗܠܝܢ ܢܘܩܙܬܐ ܕܟܘܠܡܐ ܪܒܐ ܕܘܟܬܢܝܐ ܐܘ ܢܘܩܙܬܐ ܕܟܘܠܡܐ ܙܥܘܪܐ ܕܘܟܬܢܝܐ . ܠܡܢܐ ؟

ܐܢܐ ܟܠܢܘܟ ܢܘܩܙܬܐ ܕܟܘܠܡܐ ܪܒܐ ܕܘܟܬܢܝܐ .

ܕܘܪܫܐ (٤ ـ ١)

ܐܫܟܚ ܐ. ܐܝܬ ܢܘܩܙܐ ܕܡܘܬܪܐ ܕܪ̈ܓܐ ܘܝܘܬܪܐ ܕܘܟܝܬܐ ܠܟܠ ܚܕ ܡܢ ܗܠܝܢ ܕܠܬܚܬ:

(١) $\text{ܕ}(\text{ܗ}) = \text{ܗ}^{٤} - ٣$

(٢) $\text{ܕ}(\text{ܗ}) = (\text{ܗ} + ١)^{٣}$

(٣) $\text{ܕ}(\text{ܗ}) = \text{ܗ}^{٣} - ٩\,\text{ܗ}^{٢} + ٢٤\,\text{ܗ}$

(٤) $\text{ܕ}(\text{ܗ}) = ٣ - (٢ - \text{ܗ})^{٢}$

(٥) $\text{ܕ}(\text{ܗ}) = \text{ܗ}^{٤} - ٢\,\text{ܗ}^{٢} - ٣$

(٦) $\text{ܕ}(\text{ܗ}) = ٥ + ٤\,\text{ܗ}^{٣} - \text{ܗ}^{٤}$

ܦܣܩܐ (٤ ـ ٤) ܢܘܩܙܐ ܘܡܫܡܗܬܐ ܘܢܘܩܙܐ ܕܫܘܠܛܐ:

ܢܚܙܐ ܗܘܝܐ ܣ = ܕ(ܗ) ܗܠܝܢ . ܦܘܫܩܬܐ ܕܗܘܝܐ ܡܢ [illegible]

ܣ

ܗ

ܐܣܟܡܐ (٤ ـ ٣)

ܚܙܝ ܗܘܐ ܕܗܘܡܐ ܡܢ ܡܕܝܢܬܐ ܕܐܠܐ ܟܡܣܬܓܝܐ ܘܗܘܡܐ ܐܚܪܢܐ ܢܘܩܪܐ.

ܡܬܪܨܬܐ :

(١) ܐܡܪܝܢ ܕܦܘܠܚܢܐ ܕܐܠܐ ܗܠܝܢ ܟܡܣܬܓܝܐ ܠܗ ܥܘܡܐ ܦܬܝܚܐ ܣܚܪܢܝܐ ܠܗ ܠܢܘܩܙܬܐ ܕܐܠܐ ܐ. ܗܘܐ ܢܦܠܐ ܚܘܕ ܟܠܗ ܡܢܝܢܘ ܒܐܘܐ ܥܘܡܐ

(٢) ܘܐܡܪܝܢ ܕܦܘܠܚܢܐ ܕܐܠܐ ܗܠܝܢ ܢܘܩܪܐ ܠܗ ܥܘܡܐ ܦܬܝܚܐ ܣܚܪܢܝܐ ܠܗ ܠܢܘܩܙܬܐ ܕܐܠܐ ܐ. ܗܘܐ ܢܦܠܐ ܠܬܚܬ ܟܠܗ ܡܢܝܢܘ ܠܗ ܕܐܘܐ ܥܘܡܐ.

(٣) ܢܘܩܙܬܐ ܕܫܘܠܛܐ : ܗܠܝܢ ܢܘܩܙܬܐ ܡܬܬܝܗܒܐ ܠܦܘܠܚܢܐ ܕܐܠܐ ܡܫܚܠܦ ܠܗ̇ܘ ܦܘܠܚܢܐ ܡܢ ܐܣܩܘܬܐ ܕܡܣܬܓܝܢܘܬܐ ܠܐܣܩܘܬܐ ܕܢܘܩܪܢܘܬܐ ܐܘ ܒܗܦܟܐ ܡܢ ܐܣܩܘܬܐ ܕܢܘܩܪܢܘܬܐ ܠܐܣܩܘܬܐ ܕܡܣܬܓܝܢܘܬܐ.

ܘܡܬܚܙܝܐ ܗܠܝܢ ܒܚܕܐ ܕܦܢܝܬܐ ܕܗܘܐ ܠܗܘܢ ܦܘܠܚܢܐ ܟܡܣܬܓܝܐ ܐܘ ܢܘܩܪܐ ܒܡܦܠܚܬܐ ܕܡܬܪܨܬܐ ܕܐܬܐ :

ܡܬܪܨܬܐ :

ܐ. ܗܘܝܐ ܝ = ܕ(ܣ) ܐܝܬܐ ܡܦܠܚܬܐ ܠܫܠܡܐ ܥܠ ܥܘܡܐ ܦܬܝܚܐ

(١) ܦܘܠܚܢܐ ܕ ܒܗܘܐ ܟܡܣܬܓܝܐ ܠܗ ܥܘܡܐ ܦܬܝܚܐ ܣܚܪܢܝܐ ܠܗ ܠܢܘܩܙܬܐ ܕܐܬܐ ܐ. ܗܘܝܐ ܕ(ܣ) < ܣܦܝܪܐ ܠܟܠ ܣ ܡܬܝܗܒܐ ܠܗ.

(٢) ܦܘܠܚܢܐ ܕ ܒܗܘܐ ܢܘܩܪܐ ܠܗ ܥܘܡܐ ܦܬܝܚܐ ܣܚܪܢܝܐ ܠܗ ܠܢܘܩܙܬܐ ܕܐܬܐ ܐ. ܗܘܝܐ ܕ(ܣ) > ܣܦܝܪܐ ܠܟܠ ܣ ܡܬܝܗܒܐ ܠܗ.

(٣) ܠܘܬ ܟܠ ܢܘܩܙܬܐ ܕܫܘܠܛܐ ܒܢܘܩܙܬܐ ܕܡܦܠܚܬܐ ܦܬܝܚܐ = ܣܦܝܪܐ (ܐܘ ܡܦܠܚܬܐ ܦܬܝܚܐ ܠܝܬܐ ܠܐ ܗܠܝܢ ܡܬܚܙܝܬܐ ܠܗ̇ܘ ܐܝܟܐ ܐܢܐ ܐܢܝܢ ܠܐ ܕܦܬܚܠܗ).

ܘܠܡܬܚܙܝܢܘܬܐ ܕܦܢܝܬܐ ܕܡܫܚܠܦܢܘܬܐ ܘܢܘܩܪܢܘܬܐ ܘܢܘܩܙܬܐ ܕܫܘܠܛܐ ܐ. ܐܝܟ ܡܬܚܙܝܢ ܠܡܫܚܠܦܬܐ ܕܐܬܝ :

(١) ܡܦܠܚܝܢ ܕ (ܣ)

(٢) ܦܠܚܝܢ ܕ (ܣ) = ܣܦܝܪܐ ܘܡܦܠܚܝܢ ܢܘܩܙܬܐ ܣ ܕܡܬܝܗܒܝ ܠܡܫܚܠܦܬܐ ܕܐܬܐ.

(٣) ܡܚܠܦܝܢ ܥܘܡܐ ܕ(ܣ) ܒܡܦܠܚܬܐ ܥܠ ܚܕܬܐ ܕܡܢܝܢܐ ܦܬܝܚܐ ܐܝܟ ܕܒܬܪܗ

ܒܪ̈ܬܝܢ ܘܡܦܩܢܘܬܐ ܕܡܫܪܝܬܐ ܥܠ ܗܝܕܝܢ ܦܢܝܬܐ ܕܝܬܝܪܘܬܐ ܘܒܨܝܪܘܬܐ .

(٤) ܢܘܩܙܐ ܕܡܬܚܙܝܐ ܠܥܒܕܐ ܕܠܗ ܘܦܬܓܡܐ ܒܝܢ ܦܢܝܬܐ ܕܝܬܝܪܘܬܐ ܘܒܨܝܪܘܬܐ ܥܠ ܝܠܝܗ ܢܘܩܙܐ ܕܩܘܠܒܐ ܐܝܟ ܕܥܒܕܐ ܕܠܗ ܗܘܐ ܕܟܝܢܐ .

ܦܘܫܩܐ:

ܠܟܠ ܚܕ ܡܢ ܗܠܝܢ ܥܒܕܐ ܐܚܪ̈ܢܐ ܦܢܝܬܐ ܕܝܬܝܪܘܬܐ ܘܒܨܝܪܘܬܐ ܘܢܘܩܙܐ ܕܩܘܠܒܐ ܐܢ ܐܝܬ

ܠܗ ܢܘܩܙܐ ܕܩܘܠܒܐ

(١) ܕ (ܣ) = ٢ + ٦ ܣ − ٣ ܣ^{٢}

ܕ́ (ܣ) = ٦ − ٦ ܣ

ܕ̋ (ܣ) = − ٦

ܕ̋ (ܣ) = − ٦ ≠ ܣܦܝܪܐ

ܕ̋ (ܣ) = − ٦ ܟܠܗܘܢ ܠܟܠ ܣ ∈ ܚ

∴ ܦܥܠܘܬܐ ܡܘܟܦܢܝܬܐ ܝܠܗ ܠܗ ܚ

(٢) ܕ (ܣ) = ܣ^{٣} − ١

ܕ́ (ܣ) = ٣ ܣ^{٢}

ܕ̋ (ܣ) = ٦ ܣ

ܕ̋ (ܣ) = ٦ ܣ = ٠ ⟸ ܣ = ܣܦܝܪܐ

ܕ (٠) = − ١

ܣ	٠ < ܣ	٠ > ܣ
ܪܘܫܐ ܕ̋ (ܣ)	+ + +	− − −
ܦܥܠܘܬܐ ܕܐܢܬܐ	ܝܨܘܦܬܐ	ܡܘܟܦܢܝܬܐ

∴ ܦܥܠܘܬܐ ܡܘܟܦܢܝܬܐ ܝܠܗ ܠܗ ܣ : { ܣ ∈ ܚ ، ܣ > ٠ }

ܘܝܨܘܦܬܐ ܠܗ { ܣ : ܣ ∈ ܚ ، ܣ < ٠ }

∴ (٠ ، −١) ܢܘܩܙܐ ܕܩܘܠܒܐ

(٣) ܕ (ܣ) = ܣ^{٣} − ٣ ܣ^{٢} − ٩ ܣ + ٧

$ܕ'(ܗ) = ٣ܗ^{٢} - ٦ܗ - ٩$

$ܕ''(ܗ) = ٦ܗ - ٦$

$ܕ''(ܗ) = ٦ܗ - ٦ = ٠ \Leftarrow ٦ܗ = ٦ \Leftarrow ܗ = ١$

$ܨ = ܕ(١) = ١ - ٣ - ٩ + ٧ = -٤$

ܨ	ܗ > ١	ܗ < ١
ܪܫܡܐ ܕ''(ܗ)	+ + +	− − −
ܡܥܒܕܐ ܕܐܬܐ	ܢܩܘܕܬܐ	ܡܘܗܬܢܬܐ

∴ ܡܥܒܕܐ ܡܘܗܬܢܬܐ ܗܘܐ ܠܗ ܗ : $\{ ܗ \in ܨ ، ܗ < ١ \}$

ܘܢܩܘܕܬܐ ܠܗ $\{ ܗ : ܗ \in ܨ ، ܗ > ١ \}$

∴ $(١ ، -٤)$ ܢܩܘܙܐ ܕܡܘܠܟܐ

ܕܘܪܫܐ (٤ ـ ٢)

ܠܟܠ ܚܕ ܡܢ ܗܕܐ ܕܐܠܝܢ ܡܚܝܒ ܠܝ، ܐܝܟ ܢܘܩܙܐ ܕܡܘܠܟܐ ܘܦܢܝܬܐ ܕܢܩܘܕܬܐ ܘܡܘܗܬܢܬܐ :

(١) $ܕ(ܗ) = ܗ^{٢} - ٦ܗ + ٩$

(٢) $ܕ(ܗ) = ٣ܗ - ܗ^{٣}$

(٣) $ܕ(ܗ) = (ܗ - ٢)^{٣} + ٣$

(٤) $ܕ(ܗ) = ܗ^{٣}(٤ - ܗ)$

(٥) $ܕ(ܗ) = \frac{١}{٤}ܗ^{٤} - \frac{٣}{٢}ܗ^{٢}$

ܦܢܝܬܐ (٤ ـ ٥) ܪܫܡܬܐ ܕܡܥܒܕܐ ܕܐܬܐ ܡܫܝܢܐ ܡܫܐ :

ܗܠܝܢ ܪܫܡܬܐ ܕܡܥܒܕܐ ܠܐܬܐ ܡܫܝܢܐ ܡܫܐ ܥܠ ܡܫܚܬܐ ܕܡܝܬܝܐ ܠܡܘܩܪܐ ܡܢ ܢܘܩܙܐ ܡܫܬܚܠܦܢ ܠܡܥܒܕܐ ܕܐܬܐ ܐܬܐ ܗܘܐ ܘܥܠ ܡܫܚܬܐ ܕܡܫܬܡܠܝܐ ܕܡܠܝܢܬܐ ܕܐܬܐ ܐܬܐ ܡܢ ܟܠ ܢܩܦܬܐ ܘܡܬܩܪܝܢܐ ܘܢܘܩܙܐ ܕܡܘܠܟܐ ܪܒܐ ܘܙܥܘܪܐ ܕܡܘܗܬܢܝܐ ܘܦܢܝܬܐ ܕܢܩܘܕܬܐ ܘܡܘܗܬܢܬܐ ܘܢܘܩܙܐ ܕܡܘܠܟܐ ܘܡܢ ܬܡܢ ܕܦܢܝܬܐ ܕܩܕܡܝܬܐ ܡܬܚܙܝܢ ܠܗ ܕܪܫܡܐ ܥܠ ܗܕܐ ܡܫܝܢܐ ܡܫܐ.

ܘܐܬܚܡܬܐ ܕܦܘܢܩܬܐ ܕܗܕܐ ܡܬܢܕܐ ܡܢ ܡܦܩܠܚܬܐ ܕܡܣܘܕܥܢܘܬܗ ܥܠ ܣܛܠܬܐ.

(١) ܦܣܝܚ ܢܘܩܙܬܐ ܕܠܒܝܬܐ.

(٢) ܦܣܝܚ ܦܣܩܬܐ ܕܙܘܕܐ ܘܒܨܪܬܐ ܘܡܫܚܠܦ ܐܝܟܐ ܕܢܘܩܙܬܐ ܕܠܒܝܬܐ (ܗܝܐ ܘܕܡ ܐܝܟܐ ܒܠܟ ܢܘܩܙܬܐ ܕܥܠܝܬܐ ܐܝܟ ܕܡܟܝܟܘܬܐ ܐܘ ܢܘܩܙܬܐ ܕܥܠܝܬܐ ܘܟܠܗܝ ܕܡܟܝܟܘܬܐ ܐܘ ܚܠܝܡܐ ܢܘܩܙܬܐ ܕܠܒܝܬܐ)

(٣) ܦܣܝܚ ܦܣܩܬܐ ܕܢܣܒܬܐ ܘܡܟܝܟܬܐ ܘܡܫܚܠܦ ܢܘܩܙܐ ܕܥܠܝܬܐ.

ܦܘܫܩܐ:

ܐܟܠ ܡܢ ܐܝܟ ܕܐܝܬ ܒܚܘܒ ܦܘܢܩܬܐ ܕܗܕܐ ܡܘܕܥܐ ܡܣܘܕܥܢܘܬܗ ܥܠ ܣܛܠܬܐ:

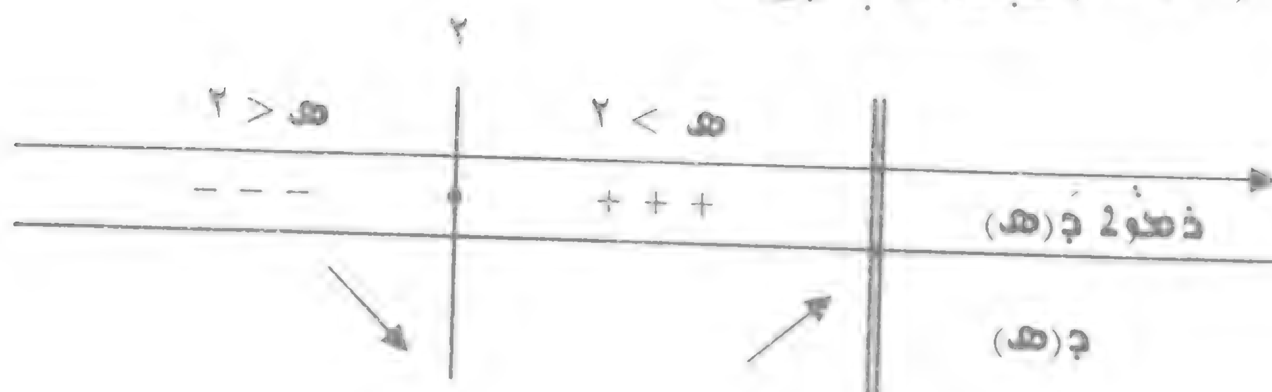
(١) $ܕ(ܣ) = ܣ^{٢} - ٤ ܣ + ٣$

$ܕ(ܣ) = ٢ ܣ - ٤$

$ܕ́(ܣ) = ٢ ܣ - ٤ = ٠ \Leftarrow ٢ ܣ = ٤ \Leftarrow ܣ = ٢$

$ܨ = ܕ(٢) = ٤ - ٨ + ٣ = -١$

∴ (٢ ، −١) ܢܘܩܙܬܐ ܕܠܒܝܬܐ



ܐܝܟܐ ܙܘܕܐ ܡܠܝܐ ܥܠ $\{ ܣ : ܣ \in ܛ ، ܣ < ٢ \}$

ܘܒܨܪܬܐ ܥܠ $\{ ܣ : ܣ \in ܛ ، ܣ > ٢ \}$

∴ (٢ ، −١) ܢܘܩܙܬܐ ܕܥܠܝܬܐ ܘܟܠܗܝ ܕܡܟܝܟܘܬܐ

$ܕ́(ܣ) = ٢$

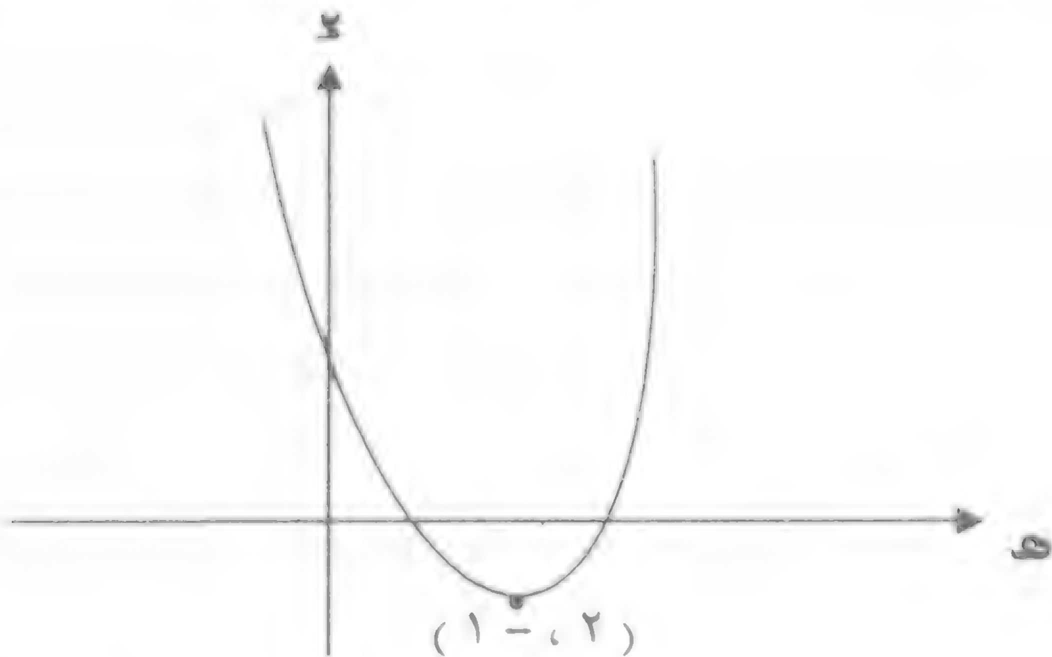
$ܕ́(ܣ) = ٢ \neq$ ܣܦܝܪܐ

∴ ܠܝܬ ܢܘܩܙܬܐ ܕܥܠܝܬܐ

$ܕ́(ܣ) = ٢$ ܦܣܝܚܐ ܠܟܠ $ܣ \in ܛ$

∴ ܦܘܢܩܬܐ ܕܗܕܐ ܝܣܘܕܐ ܡܠܝܐ ܥܠ ܛ

ܘܟܠ ܕܐܘܐ ܦܫܝܛܬܐ ܕܗܘܐ ܗܘܐ ܒܨܘܪܬܐ (٤ ـ ٤)



ܨܘܪܬܐ (٤ ـ ٤)

ܦܘܫܩܐ (٢)

$ܕ(ܗ) = ܗ^{٣} - ٣ܗ$

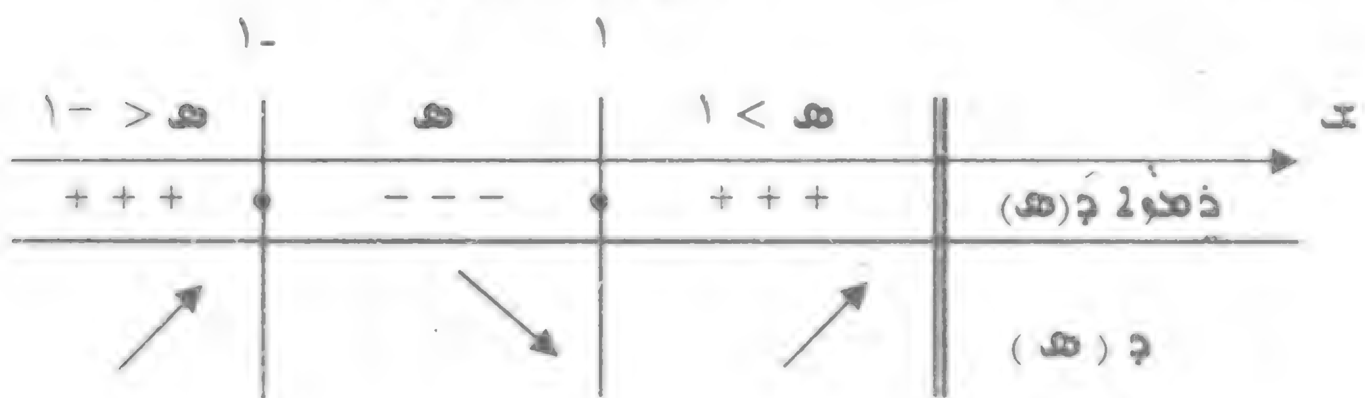
$ܕ'(ܗ) = ٢ܗ^{٢} - ٣$

$ܕ'(ܗ) = ٢ܗ^{٢} - ٣ = ٠ \Leftarrow ٢ܗ^{٢} = ٣ \Leftarrow ܗ^{٢} = ١$

ܐܢ $ܗ = ١$ ، $ܕ(١) = ١ - ٣ = -٢$

ܐܘ $ܗ = -١$ ، $ܕ(-١) = -١ + ٣ = ٢$

∴ (١ ، −٢) ، (−١ ، ٢) ܢܘܩܙܐ ܐܠܝܨܬܐ



ܚ	ܗ > ١	ܗ	ܗ < −١
ܪܫܡܐ ܕ'(ܗ)	+ + +	− − −	+ + +
ܕ(ܗ)	↗	↘	↗

ܐܢܐ ܘܡܪܝܡܬܐ ܝܠܗ ܠܗ (١) $\{ܗ : ܗ \in ܚ ، ܗ > ١\}$

(٢) $\{ܗ : ܗ \in ܚ ، ܗ < -١\}$

ܘܡܚܝܬܐ ܠܗ ܒܦܢܐ (−١ ، ١)

∴ (١ ، −٢) ܢܘܩܙܬܐ ܕܥܘܠܝܐ ܙܥܘܪܐ ܕܦܘܢܩܬܐ.

(−١ ، ٢) ܢܘܩܙܬܐ ܕܥܘܠܝܐ ܪܒܐ ܕܦܘܢܩܬܐ

$ܕ''(ܗ) = ٦ܗ$

$ܕ''(ܗ) = ٦$ = ܣܦܝܩܐ ⟸ ܗ = ܣܦܝܩܐ ، $ܕ(٠)$ = ܣܦܝܩܐ

ܣ	ܗ > ٠	ܗ < ٠
ܪ̈ܡܙܐ ܕ (ܗ)	+ + +	− − −
ܡܟܝܦܐ ܕ	ܢܫܘܦ̈ܐ	ܡܘܫܚ̈ܢܐ

ܡܟܝܦܐ ܕܐ̈ܢܐ ܡܘܫܚ̈ܢܐ ܒܠܘ ܠܗ (١) {ܗ : ܗ ∋ ܣ ، ܗ < ٠}

ܘܢܫܘܦ̈ܐ ܠܗ (٢) {ܗ : ܗ ∋ ܣ ، ܗ > ٠}

∴ (٠ ، ٠) ܗܝ ܢܘܩܙܐ ܕܫܘܚܠܦܐ

ܘܟܠ ܕ̈ܐܪܐ ܗܘ ܡܟܝܦܐ ܕܐ̈ܢܐ ܒܐܨܛܠܡܐ (٤ ـ ٥)

ܐܨܛܠܡܐ (٤ ـ ٥)

ܕܘܪܫܐ (٤ ـ ٣)

ܕܪܘܫ ܡܦܠܚܢܐ ܕܡܘܕܥܢܘܬܗ ܠܗ ܣܛܠܐ ܠܟܠ ܚܕ ܡܢ ܐ̈ܢܐ ܕܠܬܚܬ :

١) $ܕ (ܗ) = \frac{١}{٢} ܗ^{٢} + ܗ$

٢) $ܕ (ܗ) = (ܗ - ١)^{٣}$

٣) $ܕ (ܗ) = ܗ^{٢} + ٢ ܗ - ٣$

٤) $ܕ (ܗ) = ܗ^{٣} - ٣ ܗ^{٢} + ٢$

ܦܢܝܬܐ (٦ . ٤) ܡܦܠܚܢܘܬܐ ܫܘܚܠܦܢܝܬܐ ܠܥܘܡܩܐ ܙܥܘܪܐ ܘܪܒܘܪܐ ܕܦܘܢܩܫܢ :

ܠܡܫܘܚܬܐ ܕܒܪ ܐܢܫܐ ܐܝܬ ܣܘܢܩܐ ܫܘܚܠܦܢܝܬܐ . ܘܪܒܐ ܒܐ ܡܬܦܠܚ ܒ ܡܠܘܐܐ ܫܘܦܝ ܠܗܘܢ ܡܥܒܕܢܘܬܐ ܕܪܒ ܥܘܡܩܐ ܐܘ ܙܥܘܪ ܥܘܡܩܐ ܠܡܬܐ ܐܬܐ ܐܝܟ ܡܠܠܬܐ ܕܪܒ ܦܫܝܛܘܬܐ ܐܘ ܪܘܚܒܐ ܕܪܒܐ ܐܘ ܪܘܚܒܐ ܬܩܢܬܐ ܡܣܘܡ ܕܦܝܐ ܡܬܩܢܐ ܘܦܝܫ ܡܫܠܡܐ ܣܘܥܪܢܐ ܕܡܬܠܬܐ ܐܝܬ ܕܪܫܬܐ ܕܣܪܒܘܬܐ ܡܢ ܐܢܐ ܕܘܐܠܐ ܘܠܐܠܬܐ ܕܐܗܐ ܕܡܝܚ ܠܦܘܫܐ ܕܐܬܝ :

ܦܘܫܩܐ (١) :

ܐܫܟܚ ܒܘܠܟܐ ܕܪܒ ܡܬܚܡܐ ܣܚܪܗ (١٢٠) ܡ.

ܫܪܝܐ :

ܫܟܝܚ ܡܘܪܟܐ ܕܡܬܚܡܐ = ܣ

ܦܬܝܘܬܐ ܕܡܬܚܡܐ = ܨ

∴ ܦܫܝܛܘܬܐ ܦ = ܣ ܨ ... (١) (ܒܕ ܪܒ ܡܬܚܡܐ ܣܘܦܠܗ ܝܠܗ ܪܒ ܦܫܝܛܘܬܐ)

ܣܚܪܐ ܕܡܬܚܡܐ = ٢ (ܡܘܪܟܐ + ܦܬܝܘܬܐ)

١٢٠ = ٢ (ܣ + ܨ)

٦٠ = ܣ + ܨ

ܨ = ٦٠ − ܣ

ܡܫܠܦܝܢ ܡܢ ܐܢܬ ܨ ܐܝܬ ܐܣܝܪܘܬܐ (١) ܒܡܝܬ

ܦ = ܣ (٦٠ . ܣ)

ܦ = ٦٠ ܣ − ܣ٢

ܦَ = ٦٠ − ٢ ܣ = ܣܦܝܪܐ

٦٠ = ٢ܨ ⟸ ܣ = ٣٠ ܡ ܡܘܪܟܐ

ܦܬܝܘܬܐ ܨ = ٦٠ − ٣٠ = ٣٠ ܡܬܪܐ

ܘܡܬܚܡܐ ܝܠܗ ܡܪܒܥܐ ܡܢ ܫܪܝܐ ܕܠܣܬܐ ܕܒܥܝܐ ܗ.

ܦܘܫܩܐ (٢) :

ܐܫܟܚ ܡܢܝܢܐ ܕܟܢܫܗ ܥܠ ܡܪܒܥܗ ܪܒ ܡܢ ܕܗܘܐ .

ܫܪܝܐ :

ܢܬܚܫ ܠܡܢܝܢܐ = ܣ

∴ ܡܪܒܥܐ ܕܡܢܝܢܐ = ܣ²

ܘܝܬܪܐ   ܨ = ܣ − ܣ²

ܨ = ١ − ٢ ܣ

ܨ = ١ − ٢ ܣ = ٠   ١ = ٢ ܣ

ܣ = ١/٢ ܡܢܝܢܐ

ܡܘܦܬܐ (٣) :

ܐܫܟܚ ܠܬܪܝܗܘܢ ܡܢܝܢ̈ܐ ܕܟܢܘܫܝܗܘܢ (٨) ܘܟܢܘܫܐ ܕܡܪ̈ܒܥܝܗܘܢ ܙܥܘܪ ܡܐ ܕܗܘܐ

ܫܪܝܐ :

ܢܣܝܡ ܗܘܐ ܡܢܝܢܐ ܩܕܡܝܐ = ܣ

∴ ܡܢܝܢܐ ܬܪܝܢܐ = ٨ − ܣ   ܠܡܢܐ ؟

ܟܢܘܫܐ ܕܡܪ̈ܒܥܝܗܘܢ ܨ = ܣ² + (٨ − ܣ)²

ܨ = ܣ² + ٦٤ − ١٦ ܣ + ܣ²

ܨ = ٢ ܣ² − ١٦ ܣ + ٦٤

ܨ = ٤ ܣ − ١٦

ܨ = ٤ ܣ − ١٦ = ٠   ٤ ܣ = ١٦

ܣ = ٤ ܡܢܝܢܐ ܩܕܡܝܐ

٨ − ٤ = ٤ ܡܢܝܢܐ ܬܪܝܢܐ

ܡܘܦܬܐ (٤) :

ܡܢ ܡܛܫܝܐ ܣܝܡܬܐ (١٢٠) ܡܢ ܦܝܠܐ ܡܬܚܪܬܐ ܦܫܛܐ ܒܡܥܒܕܢܐ ܕܦܠܓܐ ܕܣܘܕܪܐ ܡܬܚܕܘ ܢܚܝܬ ܥܠ ܚܕ ܡܢ ܓܒ̈ܘܗܝ ܐܠܝܠܐ ܘܢܘܪܐ ܕܡܛܫܝܐ. ܡܢܐ ܢܗܘܐ ܟܘܠܢܐ ܕܐܗܐ ܡܛܫܝܐ ܕܗܘܝܐ ܠܗܘ ܫܛܝܚܘܬܐ ܦܬܝܐ ܪܒܐ ܡܬܚܕܬܐ ܟܕ ܡܐ ܕܗܘܐ.

ܫܪܝܐ :

ܢܣܝܡ ܗܘܐ ܢܘܪܐ ܕܐܠܝܠܐ ܘܢܘܪܐ ܕܡܛܫܝܐ = ٢ ܣ

ܢܘܪܐ ܕܐܠܝܠܐ ܐܚܪܢܐ = ܨ

ܫܛܝܚܘܬܐ ܕܡܛܫܝܐ = ٢ ܣ × ܨ



ܡܫܘܚܬܐ ܡܠܒܢܬܐ = ܦܠܓܗ ܕܫܘܕܪܗ ܡܠܒܢܬܐ (٢ܣ)

$$\text{ܡܫܘܚܬܐ ܡܠܒܢܬܐ} = \frac{١}{٢} \times ܣ^{٢}\ ܛ = \frac{١}{٢} \times \frac{٢٢}{٧}\ ܣ^{٢}$$

$$= \frac{٢٢}{٧}\ ܣ^{٢}$$

$$\text{ܡܫܘܚܬܐ ܦܫܝܛܬܐ ܡܠ} = \frac{١١}{٧}\ ܣ^{٢}$$

ܚܕܝܪܐ ܕܡܫܚܬܐ = ٢ (٢ܣ + ܠ) = ١٢٠

٢ܣ + ܠ = ٦٠ ⟸ | ܠ = ٦٠ − ٢ܣ |

$$ܡܠ = ٢ܣ\ (٦٠ - ٢ܣ) - \frac{١١}{٧}\ ܣ^{٢}$$

$$ܡܠ = ١٢٠ܣ - ٤ܣ^{٢} - \frac{١١}{٧}\ ܣ^{٢}$$

$$ܡܠ = ١٢٠ - ٨ܣ - \frac{٢٢}{٧}\ ܣ$$

$$ܡܠ = ١٢٠ - ٨ܣ - \frac{٢٢}{٧}\ ܣ = \text{ܣܦܝܬܐ}$$

$$٨٤٠ - ٥٦ܣ - ٢٢ܣ = ٠$$

$$٨٤٠ = ٧٨ܣ \Leftarrow ܣ = \frac{٨٤٠}{٧٨} = \frac{١٤٠}{١٣}\ \text{ܣܡ}$$

ܦܘܬܝܐ ܕܐܠܦܬܐ ܘܬܘܪܐ = ٢ܣ

$$= \frac{٢٢}{٧} \text{ ܣܡ}$$

ܡܘܪܟܐ ܕܐܠܠܐ ܠܦܘܬܐ = ܪ = ٦٠ − ٢ܗ

$$ܪ = ٦٠ - \frac{٢٢}{٧} \text{ ܣܡ}$$

$$ܪ = \frac{٧٨٠ - ٢٨٠}{١٣} = \frac{٥٠٠}{١٣} \text{ ܣܡ}$$

ܕܘܪܫܐ (٤ ـ ٤)

ܫ١ : ܐܫܟܚ ܦܬܝ ܡܪܒܥܐ ܕܩܛܪܗ (٣٠) ܘܦܬܝܘܬܐ ܕܡܘܦܩܢܗ ܐܢ ܗܘܐ ܛ ܕܗܘܐ .

ܫ٢ ܐܫܟܚ ܦܬܝ ܡܪܒܥܐ ܡܫܘܚܬܐ ܕܩܛܪܗ (٤) ܘܦܬܝܘܬܐ ܕܡܘܦܩܢܗ ܡܪܟܙܐ ܕܚܕ ܡܢܗܘܢ ܒܡܨܥܬܐ ܕܐܚܪܢܐ ܐܢ ܗܘܐ ܛ ܕܗܘܐ.

ܫ٣ ܡܬܚܐ ܕܐܪܒܥܐ ܡܫܘܚܬܐ ܕܐܫܟܢܬܐ ܐܚܪܢܝܬܐ ܢܘܪܐ ܡܢ ܚܕ ܡܢ ܦܢܝܬܗ ܐܫܟܚ ܐܢ ܬܚܘܡܐ ܡܢ ܐܪܟܐ ܡܬܝܕܥܐ ܠܗ ܘܡܬܚܗ ܒܡܬܚܐ ܗܘܪܟܗ (١٢٠) ܣܡ

ܫ٤ ܒܗܕܐ ܒܐܫܟܢܬܐ ܕܡܫܘܚܬܐ ܟܠܠ ܠܗ ܦܬܝܐ ܕܫܘܪܐ ܐܪܟܐ ܕܠܟܠ ܡܬܚܐ ܟܠ ܚܕ ܡܢ ܒܘܟܪܐ ܕܡܫܘܚܬܐ . ܐܢ ܚܕܝܐ ܕܡܫܘܚܬܐ (٨) ܣܡ ، ܐܫܟܚ ܒܘܟܪܐ ܕܡܫܘܚܬܐ ܐܢ ܕܗܘܐ ܬܚܘܡܐ ܕܒܗܕܐ ܐܢ ܛ ܕܗܘܐ .

ܫ٥ ܦܬܝܗ ܕܘܦܢܐ ܠܗܠܝܢ ܟܠܗ ܘܡܫܘܚܬܗ ܗܘܐ (ܣ) ܡܬܚܐ ܒܬܘܠܡܐ ، ܡܢ ܦܬܝܐ ܐܪܟܐ ܣ = ٢٢٤ ، − ١٦ ܘ[٢] . ܚܫܘܒ ܐܢ ܡܫܘܚܬܐ ܦܬܝܐܠܗ ܠܗܠܝܢ.

ܫ٦ ܦܢܝܐ ܠܗ ܦܠܓܘܬܐ ܕܕܝܪܬܐ ܕܢܗܪܐ ܒܐܫܟܢܬܐ ܡܫܘܚ ܡܫܘܚܬܐ ܟܠܐ ܣܡܐ ܐܝܬ ܡܪܒܥܐ ܘܦܬܝܗ (٤٨) ܡܬܚܐ ܡܫܘܚܬܐ . ܐܫܟܚ ܬܚܘܡܐ ܬܚܘܡܐ ܡܢ ܠܦܘܬ ܡܬܝܕܥܐ ܠܗ ܦܬܝܘܬܗ ܠܕܝܪܬܗ.

ܦܣܘܩܐ ܫܒܝܥܝܐ

ܓܡܝܪܘܬܐ

ܒܕܪܫ ܗܢܐ ܦܣܘܩܐ ܡܠܘܐܐ ܕܓܡܝܪܘܬܐ ܕܒܕܐܠܗ ܒܟܪܟܐ ܗܘ ܟܪܝܬܐ ܕܒܘܟܪܐ ܡܢ ܡܦܩܬܐ ܕܟܠ ܗܠܝܢ ܡܫܝܚܬܐ ܟܪܝܗܝ ܐܟܐ ܒܐܘܕܥܘ ܘܡܢ ܐܢܫܝܢ ܕܐܝܢ ܡܦܩܬܐ ܡܬܚܫܒܬܐ ܕܒܚܘܫܒܬܐ ܘܡܬܚܫܒܬܐ ܒܐܘܡܢܘܬܐ ܕܡܦܩܬܐ ܠܟܠ ܒܢܘܣܟܐ ܡܫܚܬܐ ܐ. ܦܪܢܘ ܝܕܥܬܐ ܐܝܬܐ ܕܡܟܪܝܢ ܐܝܟܐ ܕܦܩܐ ܠܗܘ ܟܠ ܢܘܣܟܐ ܡܢ ܢܘܣܘ ܐܘ ܡܬܚܫܒܬܐ ܕܫܘܫܐ ܐܘ ܡܫܝܚܘܬܐ ܠܓܘܒܝܐ ܘܚܕ ܐ. ܡܠܘܠܗ ܢܗܘܐ ܐܝܟ ܐܢܐ ܠܘܒܠܐ.

ܬܚܡܐ (٥ - ١) ܓܡܝܪܘܬܐ ܠܐ ܡܬܚܡܬܐ :

ܠܦܠܚ ܕܐ̱ܢ ܢܗܘܐ ܨ = ܬ(ܣ) ܐܝܬܐ ܡܥܠܝܬܐ ܠܫܘܠܛܐ . ܡܫܝܚܬܐ ܗܠܝܢ ܡܬܚܫܒܬܐ ܨ = ܬ(ܣ) ܒܣܘܥܪܢܘܬܐ ܦܬܝܐ ܡܕܝܢܬܐ ܫܘܠܛܐ ܗܢܐ ܘܕܝܢ ܒܣܘܥܪܢܘܬܐ ܕܫܘܠܛܐ ܡܬܚܫܒ ܐܝܬܐ ܝܕܥܬܐ ܘܗܠܝܢ ܐܝܬܐ ܚܕܬܐ

ܡܕܟܪܬܐ :

ܣܘܥܪܢܘܬܐ ܕܗܦܟܬܐ ܕܫܘܠܛܐ ܗܝ ܣܘܥܪܢܘܬܐ ܦܝܫ ܡܠܒܫܐ ܒܗ ܡܬܚܫܒܬܐ ܕܐܝܬܐ ܦܪܢܘ ܝܕܥܬܐ ܫܘܠܛܗ ܘܦܝܫ ܡܕܝܢܬܐ ܐܗܝ ܣܘܥܪܢܘܬܐ ܒܣܘܥܪܢܘܬܐ ܕܓܡܝܪܘܬܐ ܠܐ ܡܬܚܡܬܐ.

ܘܐܗܘ ܗܢܘ ܡܠܘܐܐ ܕܕܪܫܐ. ܗܢܐ ܦܣܘܩܐ ܘܡܫܝܚܬܐ ܗܠܝܢ ܡܥܕܪܬܐ ܕܓܡܝܪܘܬܐ ܠܐ ܡܬܚܡܬܐ ܒܐܣܟܡܐ :

ܦܝܫ ܐܡܝܪܐ ܠܬܐ ܬ(ܣ) ܕܗܠܝܢ ܓܡܝܪܘܬܐ (ܗܦܟܐ ܕܫܘܠܛܐ) ܕܐܬܐ ܕ(ܣ) ܠܗ ܐܘܚܕ ܦܬܝܚܐ (ܐ ، ܒ) ܐ̱ܢ ܢܗܘܐ ܬ′(ܣ) = ܕ(ܣ) ܠܟܠ ܣ ∈ (ܐ ، ܒ)

ܘܦܝܫ ܪܫܝܡܐ ܠܣܘܥܪܢܘܬܐ ܕܓܡܝܪܘܬܐ ܒܪܡܙܐ " $\int$ " ܘܒܥܒ ܕܐܝܬܐ ܐ̱ܢ ܢܗܘܐ ܬ′(ܣ) = ܕ(ܣ) ܒܐܡܪܝܢ :

$\int$ ܕ(ܣ) ܕܣ = ܬ(ܣ) + ܓ ܟܕ ܓ ∈ ܚ ܘܐܝܬܐ ܒܗܝ

(ܬ(ܣ) + ܓ)′ = ܬ′(ܣ) + ܣܦܪܐ = ܬ′(ܣ) = ܕ(ܣ)

ܦܘܫܩܐ :

ܐܬܐ	ܡܫܬܩܠܬܐ	ܫܡܥܪܝܘܬܐ
(١) $\text{ܕ}_{١}(\text{ܣ}) = \text{ܣ}^{٣}$	$\text{ܕ}'_{١}(\text{ܣ}) = ٣\,\text{ܣ}^{٢}$	$\int ٣\,\text{ܣ}^{٢}\,\text{ܕܣ} = \text{ܣ}^{٣}$
(٢) $\text{ܕ}_{٢}(\text{ܣ}) = \text{ܣ}^{٣} + \frac{٣}{٤}$	$\text{ܕ}'_{٢}(\text{ܣ}) = ٣\,\text{ܣ}^{٢}$	$\int ٣\,\text{ܣ}^{٢}\,\text{ܕܣ} = \text{ܣ}^{٣} + \frac{٣}{٤}$
(٣) $\text{ܕ}_{٣}(\text{ܣ}) = \text{ܣ}^{٣} - \sqrt{٢}$	$\text{ܕ}'_{٣}(\text{ܣ}) = ٣\,\text{ܣ}^{٢}$	$\int ٣\,\text{ܣ}^{٢}\,\text{ܕܣ} = \text{ܣ}^{٣} - \sqrt{٢}$
(٤) $\text{ܕ}_{٤}(\text{ܣ}) = \text{ܣ}^{٣} + ٥$	$\text{ܕ}'_{٤}(\text{ܣ}) = ٣\,\text{ܣ}^{٢}$	$\int ٣\,\text{ܣ}^{٢}\,\text{ܕܣ} = \text{ܣ}^{٣} + ٥$

ܘܗܟܢܐ ܡܨܝܢ ܕܐܡܪܝܢ ܠܟܠ ܓ ∈ ܚ (ܓ ܡܢܝܢܐ ܬܪܝܨܐ ܩܝܘܡܐ)

ܡܥܝܪܢܘܬܐ

١) ܪܡܙܐ ∫ ܡܫܘܐ ܥܠ ܩܘܡܪܢܘܬܐ ܕܫܡܥܪܢܘܬܐ ܠܐ ܡܬܚܡܬܐ (ܗܝ ܐܝܟ ܩܘܡܪܢܘܬܐ ܕܗܦܟܬܐ ܕܡܝܠܬܐ) ܘ ∫ ... ܣ ܩܘܡܪܢܘܬܐ ܕܦܬܐ ܩܘܡܪܢܘܬܐ ܕܦܬܐ ܚܕܝܬܐ ܥܠ ܡܫܬܠܦ ܣ ܘ ∫ . ܩܘܡܪܢܘܬܐ ܕܦܬܐ ܚܕܝܬܐ ܠܡܫܬܠܦ ، .. ܘܗܟܢܐ

٢) ܡܢ ܦܘܫܩܐ ܠܥܠ ܡܨܝܢܐ ܕܢܚܙܐ ܕܐܝܬ ܡܢܝܢܐ ܠܐ ܡܬܚܡܢܐ ܡܢ ܐܬܐ ܡܢܝܢܐ ܕܐܝܬ ܠܗܘܢ ܡܝܠܬܐ ܚܕܐ ܘܟܠ ܕܗܘܐ ، ܢܗܘܐ

$\text{ܠ}(\text{ܣ}) = \text{ܕ}(\text{ܣ}) + \text{ܓ}$

$\int \text{ܕ}(\text{ܣ})\,\text{ܕܣ} = \text{ܠ}(\text{ܣ}) + \text{ܓ}$ ܠܐ ܢܘܚܕܐ ܐܬܐ ܡܫܡܪܢܐ ܫܡܥܪܝܬܐ ܠܡܕܐ ܗܝ ܐܝܟ ܩܘܡܪܬܐ ܠܐ ܡܬܚܡܬܐ ܡܢ ܐܬܐ ܡܢܝܢܐ ܕܐܝܬ ܠܗܘܢ ܡܝܠܬܐ ܚܕܐ ܘܫܠܝܦ ܢܗܘܐ ܒܡܢܝܢܐ ܬܪܝܨܐ ܩܝܘܡܐ .

٣) ܡܢܝܢܐ ܩܝܘܡܐ ܓ ܦܪܝ ܡܢܐ ܩܝܘܡܐ ܕܫܡܥܪܢܘܬܐ ܠܐ ܡܬܚܡܬܐ ܘܥܠ ܫܡܥܪܢܘܬܐ ܠܐ ܡܬܚܡܬܐ ܩܝܘܡܐ ܡܬܚܡܐ ܠܦܘܬ ܬܪܨܐ ܢܘܗܪܐ.

ܠܘܚܐ (٥ - ٢) ܐܝܟ ܕܫܡܥܪܢܘܬܐ ܠܐ ܡܬܚܡܬܐ :

ܒܪܘܚ ܗܢܐ ܦܘܫܩܐ ܢܚܙܐ ܐܝܟ ܕܫܡܥܪܢܘܬܐ ܠܐ ܡܬܚܡܬܐ ܕܠܐ ܚܝܕܐ. ܡܫܘܚܬܗ ܘܝܠܝܦ ܐܢܘܢܐ ܕܡܫܠܡܢܘܬܐ .

> ܐܝܟ ܡܕܡܐ :
>
> (ܐ) $\int \text{ܕܣ} = \text{ܣ} + \text{ܓ}$
>
> (ܒ) $\int \text{ܣ}^{\text{ܢ}}\,\text{ܕܣ} = \frac{\text{ܣ}^{\text{ܢ}+١}}{\ldots} + \text{ܓ}$
>
> ܒܬܪܝܢ ، $\text{ܢ} \neq -١$ ، ܓ ∈ ܚ

ܦܘܫܩܐ :

ܐܫܟܚ ܟܠ ܡܢ ܗܢܐ ܕܐܬܐ :

$$(١)\ \int ܣ^{٤}\,ܕܣ = \frac{ܣ^{٤+١}}{٤+١} + ܓ = \frac{ܣ^{٥}}{٥} + ܓ$$

[ܡܫܪܝܬܐ ܗܠܝܢ ܒܡܫܚܬܐ ܕܦܠܓܘܬܐ ܐܫܘܝܬܐ ܚܕ ܕܢܐ]

$$(٢)\ \int \sqrt{ܣ}\,ܕܣ = \int ܣ^{\frac{١}{٢}}\,ܕܣ = \frac{ܣ^{\frac{٣}{٢}}}{\frac{٣}{٢}} + ܓ$$

$$= \frac{٢}{٣}\,ܣ^{\frac{٣}{٢}} + ܓ$$

$$(٣)\ \int \frac{ܕܣ}{ܣ^{٣}} = \int ܣ^{-٣}\,ܕܣ = \frac{ܣ^{-٢}}{-٢} + ܓ$$

$$= \frac{-١}{٢ܣ^{٢}} + ܓ$$

ܠܡܕܪܫܬܐ ܡܢ ܡܫܪܝܬܐ ܕܦܠܓܘܬܐ ܒܡܫܚܬܐ ܕܫܘܝܘܬܐ ܕܦܠܓܘܬܐ ܘܗܘܝܐ ܠܗܘܢܐ ܒܡܝܬܐ ܡܢܗ.

> ܐܗܐ ܡܫܪܝܬܐ :
>
> ܠܟܠ ܐ ܡܫܝܬܐ ܡܫܬܪܝܐ ܠ ܒ ܒܡܘܢܐ
>
> $$\int ܐ \times ܕ(ܣ)\,ܕܣ = ܐ \int ܕ(ܣ)\,ܕܣ$$

ܦܘܫܩܐ :

ܐܫܟܚ ܟܠ ܡܢ ܗܢܐ ܕܐܬܐ :

$$(٤)\ \int ٦\,ܣ^{٢}\,ܕܣ = ٦ \int ܣ^{٢}\,ܕܣ = ٦ \times \frac{ܣ^{٣}}{٣} + ܓ$$

$= ٢ܣ^{٣} + ܓ$ (ܡܫܪܝܬܐ ܗܠܝܢ ܒܡܫܚܬܐ ܕܦܠܓܘܬܐ ܐܫܘܝܬܐ ܚܕ ܕܢܐ)

$$(٥)\ \int \frac{١}{٢}\,ܕܣ = \frac{١}{٢} \int ܕܣ = \frac{١}{٣}\,ܣ + ܓ$$

$$(٦)\ \int \frac{٣\,ܕܣ}{٤ܣ^{٣}} = \int \frac{٣}{٤}\,ܣ^{-٣}\,ܕܣ = \frac{٣}{٤} \times \frac{ܣ^{-٢}}{-٢} + ܓ$$

$$= \frac{-3}{1.8} + \text{ܓ}$$

(٧) $\int \frac{3}{5} \sqrt[3]{\text{ܣ}}\, d\text{ܣ} = \int \frac{3}{5} \text{ܣ}^{\frac{1}{3}}\, d\text{ܣ}$

$$= \frac{3}{5} \times \frac{\text{ܣ}^{\frac{4}{3}}}{\frac{4}{3}} + \text{ܓ}$$

$$= \frac{3}{5} \times \frac{3}{4} \text{ܣ}^{\frac{4}{3}} + \text{ܓ}$$

$$= \frac{9}{20} \text{ܣ}^{\frac{4}{3}} + \text{ܓ}$$

(٨) $\int \frac{3\, d\text{ܠ}}{\sqrt{\text{ܠ}}} = \int 3\, \text{ܠ}^{-\frac{1}{2}}\, d\text{ܠ}$

$$= 3 \times \frac{\text{ܠ}^{\frac{1}{2}}}{\frac{1}{2}} + \text{ܓ} = 3 \times 2\, \text{ܠ}^{\frac{1}{2}} + \text{ܓ}$$

$$= \sqrt{\text{ܠ}} + \text{ܓ}$$

ܐܦ ܗܠܝܢ̈ܐ :

$$\int [\, \text{ܕ}(\text{ܣ}) + \text{ܦ}(\text{ܣ})\; d\text{ܣ} \,]$$

$$= \int \text{ܕ}(\text{ܣ})\, d\text{ܣ} + \int \text{ܦ}(\text{ܣ})\, d\text{ܣ}$$

ܘܡܚܝܢܐ ܗܠܝܢ ܦܫܝܛܬܐ ܒܦܘܫܩܐ ܠܗܘܐ ܐܦ ܐܢ ܕܚܝܬܐ ܒܡܬܚܘܡܐ ܕܡܘܢܝܐ ܕܡܫܝܢܐ ܠܐ ܢܛܠܩܘܢ ܡܢ ܗܢܐ ܦܪܝܫܐ ܠܒܡܬܚܘܡܐ.

ܬܘܦܣܐ :

ܐܝܟܢ ܗܠ ܡܢ :

(٩) $\int (6\text{ܣ}^2 - 5\text{ܣ} - 3)\, d\text{ܣ} =$

$$\int 6\text{ܣ}^2\, d\text{ܣ} - \int 5\text{ܣ}\, d\text{ܣ} + \int 3\, d\text{ܣ}$$

$$= 6 \frac{\text{ܣ}^3}{3} - 5 \times \frac{\text{ܣ}^2}{2} + 3\text{ܣ} + \text{ܓ}$$

$$= ٢ܣ^{٣} - \frac{٥}{٢}ܣ^{٢} + ٣ܣ + ܓ$$

ܚܙܝ ܕ ܡܫܝܢܘܬܐ ܕܠܗ ܕܟܘܢܟܬܐ ܕܩܠܝܠܐ ܐܫܬܐ ܚܕ ܕܚܕ ܐܦ ܡܫܝܢܘܬܐ ܕܠܗ ܡܫܝܢܬܐ ܕܫܘܬܦܬܐ ܟܠܗ ܒܡܬܢܝܢܘܬܐ ܠܐ ܡܬܚܫܒܬܐ ܐܠܐ ܐܦ ܒܡܢܝܢܐ ܕܐܢܘܢ ܫܘܬܦܬܐ ܚܕܐ ܫܘܬܦܬܐ . ܡܛܠ ܕܗܕܐ ܡܫܠܡܝܢ ܡܢ ܒܢܝܢܐ ܚܕ ܫܘܬܦܬܐ ܒܠܚܘܕ . ܘܡܫܡܠܝܢ ܗܕܐ ܠܗ ܐܢܝܢܘܬܐ ܕܐܬܝ :

$$(١٠)\quad \int\left(ܢ - \frac{٢}{ܢ^{٢}} + ٣\right)ܕܢ = \int\left(ܢ - ٢ܢ^{-٢} + ٣\right)ܕܢ$$

$$= \frac{ܢ^{٢}}{٢} - \frac{٢ܢ^{-١}}{-١} + ٣ܢ + ܓ$$

$$= \frac{١}{٢}ܢ^{٢} + \frac{٢}{ܢ} + ٣ܢ + ܓ$$

$$(١١)\quad \int\left(٣ܣ^{٤} - ٨ܣ^{٣} + ٥\right)ܕܣ = \frac{٢ܣ^{٥}}{٥} - \frac{٨ܣ^{٤}}{٤} + ٥ܣ + ܓ$$

$$= \frac{١}{٢}ܣ^{٥} - ٢ܣ^{٤} + ٥ܣ + ܓ$$

ܐܬܐ ܕܬܡܝܡܬܐ :

ܐܢ ܗܘܐ ܕ(ܣ) ܗܕܐ ܡܟܠܝܢܬܐ ܠܣܩܠܬܐ ܒܘܪܐ :

$$\int [ܕ(ܣ)]^{ܢ} \times ܕ'ܣ\, ܕܣ = \frac{[ܕ(ܣ)]^{ܢ+١}}{ܢ+١} + ܓ$$

ܒܟܕ $ܢ \neq -١$ ، $ܓ \in ܫ$

ܘܡܫܝܢܬܐ ܗܠܝܢ ܕܟܘܢܬܐ ܕܗܕܐ ܐܬܐ ܒܐܬܚܙܝܬܐ

$$\int ܠ^{ܢ}\, ܕܣ = \frac{ܠ^{ܢ+١}}{ܢ+١} + ܓ \quad ܟܕ\ ܠ = ܕ(ܣ)$$

، $ܢ \neq -١$ ، $ܓ \in ܫ$

حلول :

أوجد كلا من التكاملات :

(١٢) $\int (س^2 + 3س - 5)^4 (2س + 3)\, دس$

بفرض أن $ع = س^2 + 3س - 5$

$\therefore دع = (2س + 3)\, دس$

$\therefore \int (س^2 + 3س - 5)^4 (2س + 3)\, دس = \int ع^4\, دع = \frac{ع^5}{5} + ث$

$= \frac{(س^2 + 3س - 5)^5}{5} + ث$

(١٣) $\int \frac{6س\, دس}{(3س^2 + 5)^7}$

بفرض أن $ع = 3س^2 + 5 \Leftarrow دع = 6س\, دس$

$\therefore \int \frac{6س\, دس}{(3س^2 + 5)^7} = \int \frac{دع}{ع^7} = \int ع^{-7}\, دع = \frac{ع^{-6}}{-6} + ث$

$= \frac{-1}{6ع^6} + ث + \frac{1}{6(3س^2 + 5)^6} + ث$

(١٤) $\int 12(س - 3)^5\, دس$

بفرض أن $ع = س - 3 \Leftarrow دع = دس$

$\therefore \int 12(س - 3)^5\, دس = \int 12ع^5\, دع = \frac{12ع^6}{6} + ث$

$= 2ع^6 + ث = 2(س - 3)^6 + ث$

(١٥) $\int 2س \sqrt{س^2 + أ^2}\, دس$

بفرض أن $ع = س^2 + أ^2 \Leftarrow دع = 2س\, دس$

$\int 2س \sqrt{س^2 + أ^2}\, دس = \int \sqrt{ع}\, دع =$

$\int ع^{\frac{1}{2}}\, دع = \frac{ع^{\frac{3}{2}}}{\frac{3}{2}} + ث$

$$= \frac{٢}{٣} ل^{\frac{٣}{٢}} + ث = \frac{٢}{٣} (هـ^{٢} + ٤)^{\frac{٣}{٢}} + ث$$

(١٦) $\int (هـ^{٢} + ٢هـ - ٥)^{٣} (هـ + ١) \, دهـ$

نفرض أن $ل = هـ^{٢} + ٢هـ - ٥ \Leftarrow دل = (٢هـ + ٢) \, دهـ$

$\therefore دل = ٢ (هـ + ١) \, دهـ$

$\therefore \frac{١}{٢} دل = (هـ + ١) \, دهـ$

$\therefore \int (هـ^{٢} + ٢هـ - ٥)^{٣} (هـ + ١) \, دهـ = \int \frac{١}{٢} ل^{٣} \, دل$

$$= \frac{١}{٢} \times \frac{ل^{٤}}{٤} + ث = \frac{١}{٨} ل^{٤} + ث$$

$$= \frac{١}{٨} (هـ^{٢} + ٢هـ - ٥)^{٤} + ث$$

(١٧) $\int \frac{(٢هـ + ٤) \, دهـ}{(هـ + ٢)^{٥}} =$

$$\int \frac{(٢هـ + ٢) \, دهـ}{(هـ + ٢)^{٥}} = \int ٢ (هـ + ٢)^{-٤} \, دهـ$$

نفرض أن $ل = هـ + ٢ \Leftarrow دل = دهـ$

$\therefore \int ٢ (هـ + ٢)^{-٤} \, دهـ = \int ٢ ل^{-٤} \, دل = ٢ \times \frac{ل^{-٣}}{-٣} + ث$

$$= -\frac{٢}{٣} (هـ + ٢)^{-٣} + ث = \frac{-٢}{٣ (هـ + ٢)^{٣}} + ث$$

(١٨) $\int \frac{(هـ^{٢} + ١) \, دهـ}{\sqrt[٣]{٢هـ^{٣} + ٦هـ + ٥}} =$

$$\int (٢هـ^{٣} + ٦هـ + ٥)^{-\frac{٣}{٢}} (هـ^{٢} + ١) \, دهـ$$

نفرض أن $ل = هـ^{٣} + ٦ + ٥ \Leftarrow دل = (٦هـ^{٢} + ٦) \, دهـ$

$$\Leftarrow \text{د}\,\text{ل} = ٦\,(\text{هـ}^{٢} + ١)\,\text{د}\,\text{هـ}$$

$$\Leftarrow \frac{١}{٦}\,\text{د}\,\text{ل} = (\text{هـ}^{٢} + ١)\,\text{د}\,\text{هـ}$$

$$\int ٢\,(\text{هـ}^{٣} + ٦\,\text{هـ} + ٥)^{-\frac{١}{٣}}\,(\text{هـ}^{٢} + ١)\,\text{د}\,\text{هـ} = \int \text{ل}^{-\frac{١}{٣}}\,\frac{١}{٦}\,\text{د}\,\text{ل}$$

$$= \frac{١}{٦}\int \text{ل}^{-\frac{١}{٣}}\,\text{د}\,\text{ل} = \frac{١}{٦} \times \frac{\text{ل}^{\frac{٢}{٣}}}{\frac{٢}{٣}} + \text{ث}$$

$$= \frac{١}{٦} \times \frac{٣}{٢}\,\text{ل}^{\frac{٢}{٣}} + \text{ث} = \frac{١}{٤}\,\text{ل}^{\frac{٢}{٣}} + \text{ث}$$

$$= \frac{١}{٤}\,(\text{هـ}^{٣} + ٦\,\text{هـ} + ٥)^{\frac{٢}{٣}} + \text{ث}$$

$$(١٩)\quad \int \frac{٥\,\text{هـ}^{٤} + ٢\,\text{هـ}}{\text{هـ}^{٣}}\,\text{د}\,\text{هـ} = \int \text{هـ}^{-٣}\,(٥\,\text{هـ}^{٤} + ٢\,\text{هـ})\,\text{د}\,\text{هـ}$$

[illegible]

$$= \int (٥\,\text{هـ} + ٢\,\text{هـ}^{-٢})\,\text{د}\,\text{هـ} = \frac{٥}{٢}\,\text{هـ}^{٢} + \frac{٢\,\text{هـ}^{-١}}{-١} + \text{ث}$$

$$= \frac{٥}{٢}\,\text{هـ}^{٢} - \frac{٢}{\text{هـ}} + \text{ث}$$

تمارين (٥ - ١)

أوجد كلا من التكاملات الآتية :

(١) $\int \sqrt{\text{س}}\ \text{ءس}$

(٢) $\int \frac{-٤}{\sqrt[٣]{\text{س}}}\ \text{ءس}$

(٣) $\int \frac{\text{س}^{٤} - ٥\text{س}^{٢} + \text{س} - ٣}{\text{س}}\ \text{ءس}$

(٤) $\int (٣\text{س} - ٤)^{٢}\ \text{ءس}$

(٥) $\int \frac{٥\ \text{ءس}}{(٢\text{س} + ٣)^{٢}}$

(٦) $\int \frac{٤\text{س}\ \text{ءس}}{\sqrt[٣]{٢ - \text{س}}}$

(٧) $\int (\text{س} + ٣)(\text{س}^{٢} + ٦\text{س} + ١)^{\frac{٥}{٢}}\ \text{ءس}$

(٨) $\int \text{س}^{٢}(٧ - ٢\text{س}^{٣})^{٤}\ \text{ءس}$

(١٠) $\int \frac{(١ + \text{س})^{٢}}{\sqrt{\text{س}}}\ \text{ءس}$

(١١) $\int \frac{\text{س}\ \text{ءس}}{\sqrt{\text{س}^{٢} + ٩}}$

(١٢) $\int (\text{ص}^{٢} - ٣)(\text{ص} + ٤)\ \text{ءص}$

(١٣) $\int \frac{\text{ص}^{٤} - ١٦}{\text{ص}^{٢} - ٤}\ \text{ءص}$

(١٤) $\int (\text{ص}^{٣} + \frac{٥}{\text{ص}^{٤}})\ \text{ءص}$

(١٥) $\int (٣\text{ص}^{\frac{١}{٣}} + ١)^{٢}\ \text{ءص}$

(١٦) $\int ٦\sqrt{\text{ص} + ١}\ \text{ءص}$

(١٧) $\int ٤٥(٤\text{س}^{٢} - ١)\sqrt[٣]{٤\text{س}^{٣} - ٣\text{س} + ١}\ \text{ءس}$

(١٨) $\int \frac{(٢ + ٣\text{س})\ \text{ءس}}{\sqrt{٢ - ٤\text{س} - ٣\text{س}^{٢}}}$

ܦܸܢܕܵܐ (٥ . ٢) ܡܦܲܠܚܵܢܘܼܬܵܐ ܥܲܠ ܓܡܝܼܪܘܼܬܵܐ ܠܵܐ ܡܚܲܕܕܬܵܐ

ܬܸܒܥܝܼ ܒܗܵܕܵܐ ܦܸܢܕܵܐ ܥܲܠ ܬܪܹܝܢ ܡܦܲܠܚܵܢܘܼܬܵܐ ܓܸܘܡܸܛܪܵܝܬܵܐ ܘܚܲܕܟܡܵܐ ܡܦܲܠܚܵܢܘܼܬܵܐ ܦܝܼܙܝܼܩܵܝܬܵܐ .

ܡܦܲܠܚܵܢܘܼܬܵܐ ܓܸܘܡܸܛܪܵܝܬܵܐ :

ܝܕܝܥܐ ܒܩܪܒܐ ܕܓܡܝܪܘܬܐ ܝܠܗ ܦܘܫܪܘܬܐ ܕܦܘܠܚܢܐ ܠܣܛܠܢܐ ܘܝܕܝܥܐ ܕܣܛܠܢܐ ܕܟܝܦܐ ܐܝܟ ܢܘܩܙܐ ܡܚܪܒܐ ܦܘܠܚܢܐ ܝܠܗ ܦܠܐ ܕܡܚܬܬܐ ܠܟܝܦܐ ܕܐܢܐ ܠܗܘ ܢܘܩܙܐ ܘܥܠ ܕܐܗܐ :

ܡܲܬܠܵܐ (١) :

ܐܫܟܚ ܦܘܠܚܢܐ ܕܟܝܦܐ ܕܐܢܐ ܠܗܘ ܟܠ ܢܘܩܙܐ ܗܘܐ (٤ܣ + ١) ܘܥܒܪ ܒܢܘܩܙܐ (−١ ، ٣)

ܫܪܵܝܵܐ :

$ܕ'(ܣ) = ٤ܣ + ١$

$\therefore ܕ(ܣ) = \int (٤ܣ + ١)\, ܕܣ = ٢ܣ^{٢} + ܣ + ܓ$

ܘܟܕ ܟܝܦܐ ܥܒܪ ܒܢܘܩܙܐ (−١ ، ٣) ܒܗܘܐ :

$٣ = ٢ - ١ + ܓ$

$ܓ = ٢$

ܒܗܘܐ : $ܕ(ܣ) = ٢ܣ^{٢} + ܣ + ٢$ ܦܘܠܚܢܐ ܕܟܝܦܐ

ܡܲܬܠܵܐ (٢)

ܐܢܐ ܣܛܠܢܐ ܕܟܝܦܐ (٦ܣ − ٤) ܐܝܟ ܢܘܩܙܐ ܐܠܝܨܬܐ (١ ، ٥) . ܐܫܟܚ ܦܘܠܚܢܐ ܕܟܝܦܐ ܕܐܗܐ

ܫܪܵܝܵܐ :

$ܕ'(ܣ) = ٦ܣ - ٤$

$ܕ(ܣ) \int (٦ܣ - ٤)\, ܕܣ$

$ܕ(ܣ) = ٣ܣ^{٢} - ٤ܣ + ܓ$

ܘܟܕ ܝܠܗ ܢܘܩܙܐ ܐܠܝܨܬܐ ܠܗܘ $ܣ = ١$

ܒܗܘܐ $ܕ(ܣ) =$ ܣܦܝܩܐ ܐܝܟ $ܣ = ١$

$\therefore$ ܣܦܝܩܐ $= ٣ - ٤ + ܓ \Leftarrow$ $\boxed{ܓ = ١}$

∴ ܕ′ (ܗ) = ٣ ܗ² − ٤ ܗ + ١

∴ ܕ (ܗ) = ∫ (٣ ܗ² − ٤ ܗ + ١) ܕܗ

ܕ (ܗ) = ܗ³ − ٢ ܗ² + ܗ + ܓ

ܘܟܕ (١ ، ٥) ܒ ܚܡܝܠܐ ܕܐܠܐ

٥ = ١ − ٢ + ١ + ܓ ⟸ ܓ = ٥

ܕ (ܗ) = ܗ³ − ٢ ܗ² + ܗ + ٥

ܡܦܠܚܢܘܬܐ ܦܝܣܝܩܝܬܐ :

ܐܢ ܗܘܐ ܫ = ܕ(ܘ) ܟܕ ܫ ܡܘܫܚܘܬܐ ܕܚܘܪܟܐ ܐܝܟ ܚܘ ܐܝܟ ܚܝܠܐ ܠܙܒܢܐ ܕܘܒܪܐ ܒܩܠܝܠܘ ܘ ܒܪܘܫܐ :

ܡܠܘܠܘܬܐ ܡܙܝܢܬܐ ܫ(ܘ) = ܕ ܫ / ܕ ܘ = ܫ′

ܘܡܘܙܥܙܥܢܐ ܒܒܠ ܙܒܢܐ ܗ(ܘ) = ܕ² ܫ / ܕ ܘ² = ܫ″

ܡܢ ܡܡܠܐܒܘܬܐ ܘܦܚܡܐ ܕܫܩܠܬܐ ܒܪܘܫܐ :

ܫ = ∫ ܗ (ܘ) ܕ ܘ ، ܗ (ܘ) = ∫ ܫ (ܘ) ܕ ܘ

ܠܘܦܐ (١)

ܠܘܚܡܐ ܐܝܟ ܚܘ ܚܝܠܐ ܠܙܒܢܐ ܒܢ ܡܠܘܠܘܬܗ ܡ ܒܫܢܐ . ܘܦܦܐ ܡܢ ܥܘܕܢܐ ܕܙܘܒܢܗ ܗܠܘ ܡ = ٤ܘ + ٢٤ . ܐܫܟܚ ܡܘܫܚܘܬܗ ܒܒܠ ܙܒܢܐ

ܠܡ ܐܝܕܐ ܕܡܘܫܚܘܬܗ

= ܡܦܝܬܐ ܠܗ ܥܘܕܢܐ ܕܙܘܒܢܐ.

ܥܒܕܐ : ܫ′ = ܗ = ٤ ܘ + ٢٤

ܫ = ∫ (٤ ܘ + ٢٤) ܕ ܘ

ܫ = ٢ ܘ² + ٢٤ ܘ + ܠ

ܐܢܗ ܘ = ٠ ، ܫ = ܡܦܝܬܐ ⟸ ܠ = ܡܦܝܬܐ ؟

∴ ܫ = ٢ ܘ² + ٢٤ ܘ

ܡܘܦܬܐ (٢)

ܓܘܫܡܐ ܙܝܥ ܥܠ ܚܛܐ ܬܪܝܨܐ ܐܝܟܐ ܕܗܘܐ ܡܣܪܗܒܢܘܬܗ ܒ ܙܒܢܐ ܢ ، ܪܦܦܐ ܗܘܐ (٦ܙ + ٤) ܘܐܢ ܡܪܗܛܢܘܬܗ ܒܙܒܢܐ ٣ ܡ/ܪ . ܐܫܟܚ ܡܪܗܛܢܘܬܗ ܘܫܘܦܗ ܒܬܘܠܡܐ (٤) ܪܦܦܐ ܡܢ ܫܘܪܝܐ ܕܙܘܥܗ.

ܫܪܝܐ :

ܡܪܗܛܢܘܬܐ ܣ(ܙ) = ٦ ܙ + ٤

∴ ܡܣܪܗܒܢܘܬܐ ܗ(ܙ) = ∫ (٦ܙ + ٤) ܕܙ = ٣ܙ² + ٤ ܙ + ܓ

ܐܢܟܝ ܙ = ܗܩܝܡܐ ، ܗ = ٣ ܒܗܝܢܐ ܒܫܘܪܝܐ

٣ = ٠ + ٠ + ܓ ⇐ ܓ = ٣

ܗ (ܙ) = ٣ܙ² + ٤ ܙ + ٣

ܡ = ∫ (٣ܙ² + ٤ ܙ + ٣) ܕܙ = ܙ³ + ٢ܙ² + ٣ ܙ + ܟ

ܐܢܟܝ ܙ = ٠ ، ܡ = ٠ ⇐ ܟ = ܗܩܝܡܐ

ܡ = ܙ³ + ٢ܙ² + ٣ ܙ ܘܐܢܟܝ ܙ = ٤

ܗ = ٣ × ١٦ + ٤ × ٤ + ٣ = ٧٦ ܡ/ܪ

ܡ = ٦٤ + ٣٢ + ١٢ = ١٠٨ ܡ ܡܘܫܚܘܬܗ

ܕܘܪܫܐ (٥ ـ ٢)

ܬ١ : ܐܫܟܚ ܡܫܘܘܝܬܐ ܕܟܝܦܐ ܕܟܢܫܗ ܠܗ̇ ܒܟܠ ܢܘܩܙܐ ܡܢ ܢܘܩܙܗ ܝܠܗ (٣ܗ² −٢ܗ +٤) ܘܥܒܪܐ ܒܢܘܩܙܐ (٢ ، −٣)

ܬ٢ : ܗܿܝ ܡܫܘܘܝܬܗ ܕܟܢܫܗ (٦ܗ) ܐܝܬܠܗ ܢܘܩܙܐ ܕܒܘܠܡܐ ܪܒܐ ܕܒܘܩܝܢܐ ܝܠܗ (−١ ، ٤) . ܐܫܟܚ ܡܫܘܘܝܬܗ.

ܬ٣ : ܓܘܫܡܐ ܙܝܥ ܥܠ ܚܛܐ ܒܡܪܗܛܢܘܬܐ ܣ(ܙ) = (٦ܙ² + ٤ܙ + ٥) ܫܚܕܘܬܐ ܒܟܠ ܙܒܢܐ ܙ . ܐܫܟܚ ܡܘܫܚܘܬܗ ܒܟܠ ܙܒܢܐ ܙ.

ܬ٤ : ܓܘܫܡܐ ܙܝܥ ܥܠ ܚܛܐ ܬܪܝܨܐ ܒܡܣܪܗܒܢܘܬܐ ܗܘܐ ٥ ܡ/ܪ² ܐܫܟܚ ܡܘܫܚܘܬܗ ܒܟܠ ܙܒܢܐ ܐܢ ܦܣܩܠܗ ܡܫܚܬܐ ܕܗܘܐ ܚܬܝܬܐ ٦ ܪܦܦܐ ܒܗܘܐ ܥܠ ܒܡܫܚܬܐ ١٨٠ ܡ ܡܢ ܢܘܩܙܐ ܕܫܘܪܝܐ ܘܡܪܗܛܢܘܬܗ ܒܗܝ ܪܦܦܐ (٤٥) ܡ/ܪ.

هەرێمی کوردستانی عێراق
وەزارەتی پەروەردە

ܡܠܟܬܐ ܕܟܘܪܕܣܬܢ . ܥܝܪܩ

ܘܙܝܪܘܬ ܡܪܕܘܬܐ

هەرێمی کوردستانی عیراق

وەزارەتی پەروەردە

ܡܢܝܢܘܬܐ

ܣܦܪܐ ܫܬܝܬܐ ܣܦܪܝܬܐ

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